Problem 15.1

* Find the Laplace transform of: *

a) cosh at

* $\cosh x = \frac{1}{2}(e^{x} + e^{-x})$

$\cosh at = \frac{1}{2}(e^{at} + e^{-at})$

$= \frac{1}{2}\left(\frac{1}{s-a} + \frac{1}{s+a}\right)$

$= \frac{1}{2}\left(\frac{(s+a)+(s-a)}{(s-a)(s+a)}\right) = \frac{1}{\cancel{2}}\left(\frac{\cancel{2}s}{s^2-a^2}\right) = \boxed{\frac{s}{s^2-a^2}}$ ✓

b) sinh at

* $\sinh x = \frac{1}{2}(e^{x} - e^{-x})$

$\sinh at = \frac{1}{2}(e^{at} - e^{-at})$

$= \frac{1}{2}\left(\frac{1}{s-a} - \frac{1}{s+a}\right) = \frac{1}{2}\left(\frac{(s+a)-(s-a)}{s^2-a^2}\right)$

$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}a}{s^2-a^2}\right)$

$= \boxed{\frac{a}{s^2-a^2}}$ ✓

Prove the following

Problem 15.40

$$L^{-1}\left[\frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}\right] = \left[\sqrt{2}e^{-t}\cos(2t+45^\circ)+3e^{-2t}\right]u(t)$$

$$F(s) = \frac{4s^2+7s+13}{(s+2)(s^2+2s+5)}$$

$$= \frac{A}{s+2} + \frac{Bs+C}{s^2+2s+5}$$

used residue method

$$A = (s+2)F(s)\Big|_{s=-2} = \frac{4s^2+7s+13}{s^2+2s+5}\Big|_{s=-2} = \frac{15}{5} = 3$$

If $s=0$:

$$F(0) = \frac{13}{10} = \frac{A}{2} + \frac{C}{5}$$

$$13 = 5A + 2C$$

Since $A=3$ solve for C:

$$13-15 = 2C$$

$$C = -1$$

If $s=1$, solve for B:

$$\frac{24}{24} = \frac{A}{3} + \frac{B+C}{8}$$

$$24 = 8A + 3B + 3C$$

$$24 = 8\cdot 3 + 3B + 3\cdot -1$$

$$0 = 3B - 3$$

$$B = 1$$

$A=3,\ B=1,\ C=-1$

Complex Poles

pg 692

$$A_1 s + A_2 = A_1(s+\alpha) + B_1\beta$$

Therefore, $H(s) = \dfrac{3}{s+2} + \dfrac{s-1}{s^2+2s+5}$

$$s^2 + as + b = s^2 + 2\alpha s + \alpha^2 + \beta^2 = (s+\alpha)^2 + \beta^2$$

$$= \frac{3}{s+2} + \frac{1(s+1) + (-1)\cdot 2}{s^2 + 2\cdot 1\cdot s + 1^2 + 2^2}$$

$$= \frac{3}{s+2} + \frac{1(s+1) + (-1)\cdot 2}{(s+1)^2 + 2^2}$$

$$F(s) = \frac{A_1(s+\alpha)}{(s+\alpha)^2+\beta^2} + \frac{B_1\beta}{(s+\alpha)^2+\beta^2}$$

$$= \frac{3}{s+2} + \frac{s+1}{(s+1)^2+2^2} + \frac{(-1)2}{(s+1)^2+2^2}$$

$$h(t) = \left(3e^{-2t} + e^{-t}\cos 2t - 1\, e^{-t}\sin 2t\right) u(t)$$

$$h(t) = \left(3e^{-2t} + Re^{-t}\cos(2t-\theta)\right) u(t)$$

$$R = \sqrt{(1)^2 + (-1)^2} = \sqrt{2} \qquad \theta = \tan^{-1}\frac{(-1)}{1} = -45^\circ$$

Therefore $h(t) = \left(3e^{-2t} + \sqrt{2}\, e^{-t}\cos(2t - (-45^\circ))\right) u(t)$

$$= \left[3e^{-2t} + \sqrt{2}\, e^{-t}\cos(2t + 45^\circ)\right] u(t)$$

Initial conditions: $v(0) = v'(0) = v''(0)$

use table on pg 687 ('time differentiation')

solve for y(t) in the following diff. eq. if the initial condition are zero

Problem 15.55

$\to \frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 8\frac{dy}{dt} = e^{-t}\cos 2t$

$$\left[s^3Y(s) - s^2\cancel{y(0)} - s\cancel{y'(0)} - \cancel{y''(0)}\right] + 6\left[s^2Y(s) - s\,y(0) - y'(0)\right] +$$

$$8\left[sY(s) - \cancel{y(0)}\right] = \frac{s+1}{(s+1)^2 + 2^2}$$

$$s^3Y(s) + 6s^2Y(s) + 8sY(s) = \frac{s+1}{(s+1)^2+2^2}$$

$$(s^3 + 6s^2 + 8s)\,Y(s) = \frac{s+1}{(s+1)^2+2^2}$$

$$s(s^2+6s+8)\,Y(s) = \frac{s+1}{(s^2+2s+5)}$$

$$\frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+4} + \frac{Ds+E}{s^2+2s+5} = Y(s) = \frac{s+1}{s(s+2)(s+4)(s^2+2s+5)}$$

$$= sY(s)\Big|_{s=0} = \frac{s+1}{(s+2)(s+4)(s^2+2s+5)}\Big|_{s=0} = \frac{1}{40}$$

$$= (s+2)Y(s)\Big|_{s=-2} = \frac{s+1}{s(s+4)(s^2+2s+5)}\Big|_{s=-2} = \frac{1}{20}$$

$$= (s+4)\,Y(s)\Big|_{s=-4} = \frac{s+1}{s(s+2)(s^2+2s+5)}\Big|_{s=-4} = \frac{-3}{104}$$

$D = -3/65$ and $E = -7/65$

If $s=0$

$$Y(s) = \frac{1}{40s} + \frac{1}{20(s+2)} - \frac{3}{104(s+4)} + \frac{-\frac{3s}{65} - \frac{2}{65}}{s^2+2s+5} = \frac{s+1}{s(s+2)(s+4)(s^2+2s+5)}$$

$$Y(s) = \frac{1}{40s} + \frac{1}{20(s+2)} - \frac{3}{104(s+4)} \underbrace{\left[\frac{3s}{65[(s+1)^2+2^2]} - \frac{7}{65[(s+1)^2+2^2]} \right]}_{Y_1(s)}$$

$$y(t) = \frac{1}{40} + \frac{1}{20}e^{-2t} - \frac{3}{104}e^{-4t} + $$

$$Y_1(s) = \frac{\frac{-3}{65}s - \frac{7}{65}}{(s+1)^2+2^2} = \frac{\frac{-3}{65}(s+1) - \frac{4}{65}}{(s+1)^2+2^2}$$

$$Y_1(s) = \frac{-3/65(s+1)}{(s+1)^2+2^2} + \frac{\frac{-2}{65}\cdot 2}{(s+1)^2+2^2}$$

$$y(t) = \left[\frac{1}{40} + \frac{1}{20}e^{-2t} - \frac{3}{104}e^{-4t} - \frac{3}{65}e^{-t}\cos(2t) - \frac{2}{65}e^{-t}\sin(2t)\right]u(t)$$