1

Answers

A1.

The matrix form is

$$\begin{bmatrix} 3 & -1 \\ -6 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \end{bmatrix}$$

The determinants are

$$\Delta = \begin{vmatrix} 3 & -1 \\ -6 & 18 \end{vmatrix} = 3 \times 18 - (-1)(-6) = 48$$

$$\Delta_1 = \begin{vmatrix} 4 & -1 \\ 16 & 18 \end{vmatrix} = 4 \times 18 - (-1)(16) = 88$$

$$\Delta_2 = \begin{vmatrix} 3 & 4 \\ -6 & 16 \end{vmatrix} = 3 \times 16 - (4)(-6) = 72$$

Hence:

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{88}{48} \fallingdotseq \boxed{1.833} \qquad x_2 = \frac{\Delta_2}{\Delta} = \frac{72}{48} = \boxed{1.5}$$

A2

The matrix form is

$$\begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix}$$

$$D = \begin{vmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{vmatrix} = \begin{array}{c|ccc|c} & 3 & -1 & -2 & \\ & -1 & 6 & -3 & \\ & -2 & -3 & 6 & \\ - & 3 & -1 & -2 & + \\ - & -1 & 6 & -3 & + \\ - & & & & + \end{array}$$

$$= 3\times6\times6 + (-3)(-1)(-2) + (-2)(-1)(-3)$$

$$-(-2)6(-2) - (-3)(-3)3 - 6(-1)(-1)$$

$$= 108 - 6 - 6 - 24 - 27 - 6 = 39$$

$$D_1 = \begin{vmatrix} 1 & -1 & -2 \\ 0 & 6 & -3 \\ 6 & -3 & 6 \end{vmatrix} = \begin{array}{c|ccc|c} & 1 & -1 & -2 & \\ & 0 & 6 & -3 & \\ & 6 & -3 & 6 & \\ - & 1 & -1 & -2 & + \\ - & 0 & 6 & -3 & + \\ - & & & & + \end{array}$$

$$= 1\times6\times6 + 0(-3)(-2) + 6(-1)(-3) - (-2)(6)(6) - (-3)(-3)1$$

$$-6(-1)0 \quad = 36 + 0 + 18 + 72 - 9 = 117$$

$$D_2 = \begin{vmatrix} 3 & 1 & -2 \\ -1 & 0 & -3 \\ -2 & 6 & 6 \end{vmatrix} = \begin{array}{c|ccc|c} & 3 & 1 & -2 & \\ & -1 & 0 & -3 & \\ & -2 & 6 & 6 & \\ - & 3 & 1 & -2 & + \\ - & -1 & 0 & -3 & + \\ - & & & & + \end{array}$$

$$= 0 + (-1)(6)(-2) + (-2)(-3) - 0 - (-3)6(3) - 6(1)(-1)$$

$$= 0 + 12 + 6 - 0 + 54 + 6 = 78$$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 1 \\ -1 & 6 & 0 \\ -2 & -3 & 6 \end{vmatrix} = \begin{array}{c|ccc|c} & 3 & -1 & 1 & \\ & -1 & 6 & 0 & \\ & -2 & -3 & 6 & + \\ - & 3 & -1 & 1 & + \\ - & -1 & 6 & 0 & + \\ - & & & & \end{array}$$

$$= 3(6)(6) + (-1)(-3) + 0 - 6(-2) - 0 - 6(-1)(-1)$$

$$= 108 + 3 + 12 - 6 = 117$$

$$x_1 = \frac{\Delta_1}{\Delta} = \frac{117}{39} = \boxed{3}$$

$$x_2 = \frac{\Delta_2}{\Delta} = \frac{78}{39} = \boxed{2}$$

$$x_3 = \frac{\Delta_3}{\Delta} = \frac{117}{39} = \boxed{3}$$

$\boxed{A3}$

$$\begin{cases} 2y_1 - y_2 = 4 \\ y_1 + 3y_2 = 9 \end{cases} \qquad \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \end{bmatrix}$$

$$A\cancel{X}Y = B \qquad A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2\times 3 - (-1)} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$Y = A^{-1}B = \frac{1}{7} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\Rightarrow \boxed{\begin{matrix} y_1 = 3 \\ y_2 = 2 \end{matrix}}$$

A4

$y_1 - y_3 = 1$

$2y_1 + 3y_2 - y_3 = 1$

$y_1 - y_2 - y_3 = 3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$AY = B \rightarrow Y = A^{-1}B$

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 1 & -1 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$c_{11} = \begin{vmatrix} 3 & -1 \\ -1 & -1 \end{vmatrix} = -3 - 1 = -4 \qquad c_{12} = -\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix} = -[-2 - (-1)] = 1$

$c_{13} = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = -2 - 3 = -5 \qquad c_{21} = -\begin{vmatrix} 0 & -1 \\ -1 & -1 \end{vmatrix} = 1$

$c_{22} = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = -1 + 1 = 0 \qquad c_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} = -[-1 - 0] = 1$

$c_{31} = \begin{vmatrix} 0 & -1 \\ 3 & -1 \end{vmatrix} = 0 - (-3) = 3 \qquad c_{32} = -\begin{vmatrix} 1 & -1 \\ -2 & -1 \end{vmatrix} = -1$

$c_{33} = \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 3$

$|A| = [c_{11} + 0 + (-1)\, c_{13}] = -4 + 0 + 5 = 1$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 0 & 1 \\ 3 & -1 & 3 \end{bmatrix}$$

$$C^T = \begin{bmatrix} -4 & 1 & 3 \\ 1 & 0 & -1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -4 & 1 & 3 \\ 1 & 0 & -1 \\ -5 & 1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 5 \end{bmatrix}$$

$$\boxed{y_1 = 6 ,\ y_2 = -2,\ y_3 = 5}$$

B1 a) $z_1 = 3 - j4$ $\quad r_1 = \sqrt{3^2+4^2} = 5$, $\quad \theta_1 = 360° - \tan^{-1}\frac{4}{3} = 306.9°$

$z_1 = \boxed{5 \angle 306.9°}$ $\quad z_1 = \boxed{5e^{j306.9°}}$

b) $z_2 = 5 + j12$ $\quad r_2 = \sqrt{5^2+12^2} = 13$ $\quad \theta_2 = \tan\frac{12}{5} = 67.38°$

$z_2 = \boxed{13\angle 67.38°}$ $\quad z_2 = \boxed{13e^{j67.38°}}$

c) $z_3 = -3 - j9$ $\quad r_3 = \sqrt{(-3)^2+(-9)^2} = 9.487$

$\theta_3 = 180° + \tan^{-1}(\frac{9}{3}) = 251.6°$

$z_3 = \boxed{9.487\angle 251.6°}$ $\quad z_3 = \boxed{9.487e^{j251.6°}}$

d) $z_4 = -7 + j$ $\quad r_4 = \sqrt{7^2+1^2} = 7.071$ $\quad \theta_4 = 180° - \tan^{-1}(\frac{1}{7})$

$= 171.9°$

$z_4 = \boxed{7.071\angle 171.9°}$ $\quad z_4 = \boxed{7.071e^{j171.9°}}$

B2 (a) $-8\angle 210° = -8\cos(210°) - j8\sin(210°) = \boxed{6.928 + j4}$

(b) $40\angle 305° = 40\cos(305°) - j40\sin(305°) = \boxed{22.94 - j32.77}$

(c) $10e^{-j30°} = 10[\cos(30°) - j\sin(30°)] = \boxed{8.66 - j5}$

(d) $50e^{j\pi/2} = 50[\cos(\pi/2) - j\sin(\pi/2)] = \boxed{j50}$

B3 $C = -3 + j7 \quad D = 8 + j$,

$C^* = -3 - j7 \quad D^* = 8 - j$

(a) $(C - D^*)(C + D^*) = [(-3+j7) - (8-j)][(-3+j7) + (8-j)]$

$= [-11 + j8][5 + j6] = (-11)5 + j(-11)(6) + j40 - 48$

$= -103 - j26$

(b) $D^2/C^* = \dfrac{(8+j)(8+j)}{(-3-j7)} = \dfrac{(63+16j)(-3+j7)}{9+49}$

$= \dfrac{-301+j393}{58} = \boxed{-5.19 + j6.776}$

(c) $2CD/(C+D) = \dfrac{2(-3+j7)(8+j)}{[(-3+j7)+(8+j)]} = 6.04 + j11.53$

B4 (a) $6\angle 30^\circ = 6\cos(30^\circ) + j6\sin(30^\circ) = 5.196 + j3$

$2e^{j45^\circ} = 2\cos(45^\circ) + j2\sin(45^\circ) = 1.414 + j1.414$

$\dfrac{5.196 + j3 + j5 - 3}{-1 + j + 1.414 + j1.414} = \cancel{(2.20+j8)(0.41-j2.41)}$

$= \dfrac{2.196 + j8}{0.414 + j2.414} = \dfrac{8.296\angle 74.65^\circ}{2.449\angle 80.26} = 3.887\angle -5.615^\circ$

(b) $15 - j7 = \sqrt{15^2+7^2}\ \angle 360° - \tan^{-1}(\frac{7}{15}) = 16.6\ \angle 335°$

$(3+j2)^* = 3 - j2 = \sqrt{3^2+2^2}\ \angle 360° - \tan^{-1}(\frac{2}{3}) = 3.61\ \angle 326°$

$(4+j6)^* = \sqrt{4^2+6^2}\ \angle 360° - \tan^{-1}(\frac{6}{4}) = 7.21\ \angle 304°$

$$\left[\frac{(16.6\angle 335°)(3.61\angle 326°)}{(7.72\angle 304°)(3\angle 70°)}\right]^* = \left[\frac{59.9\angle 301°}{21.6\angle 14°}\right]^* = (2.77\angle 287$$

$= 2.77\angle -287°$

$\boxed{B5}$ (a) $|A| = \sqrt{3^2+4^2} = 5 \qquad \theta = 360° - \tan^{-1}(\frac{4}{3})\ ,\ A = 5\angle 307°$

$A^{1/3} = 5^{1/3}\ \angle 307/3 + 2\pi K/3\ ,\ K = 0, 1, 2$

$$\Rightarrow \boxed{\begin{aligned} A^{1/3} &= 1.71\angle 102.3° \\ A^{1/3} &= 1.71\angle 222.3° \\ A^{1/3} &= 1.71\angle 342.3° \end{aligned}}$$

(b) $A = 5\angle 307°, \qquad A = 5e^{j307°}$

$\ln A = \ln(5e^{j307°})\ \cancel{= \ln(5) + j307°}$

$= 1.609 + j5.356 + j2n\pi \quad (n = 0, 1, 2, \dots)$