# Chapter 5 – Machine Models

## *5.1   Introduction*

Machine analysis looks at the energy conversion segment of electrical systems. The forms of energy fields are electrical, magnetic, and mechanical which are orthogonal. A single coil conversion is either to frequency or to mechanical. Two coils form a machine.

Machines can be modeled as a Thevenin equivalent voltage and impedance with a magnetizing circuit consisting of an inductor with its resistance. The magnetizing inductor induces flux which produces the Thevenin voltage. Three types of problems are encountered.

1)  Model parameters and losses require the complete model using circuit theory.

2)  Transients and load flow use the Thevenin equivalent.

3)  Steady state uses the terminal conditions with complex power.

The time domain signal representation contains all the components of the responses including DC, transient, and sinusoidal.

$$y(t) = F + (I - F)e^{-t/\tau} \cos(\omega t + \theta)$$

## *5.2   Coils*

Magnetic devices are made of coils of wire, called windings. As a circuit element, they are an inductor. Since there is copper in the winding, there will also be a resistance. Devices consist of one coil, while machines contain two coils. Three-phase devices have three coils on the input and three on the ouput.

### 5.2.1  Single coil

An earlier chapter looked at the simplest magnetic device, a single coil of wire constructed on a magnetic core to constrain the magnetic field. These magnetic devices can have several functions. In most cases the inductors in a circuit are limited to applications that transfer energy to another form.

Inductors are inherently larger than other electrical elements. They are also complex, since they require the winding of wire about a core. Other electrical elements can be created using integrated circuit technology. At this time, inductors cannot be integrated.
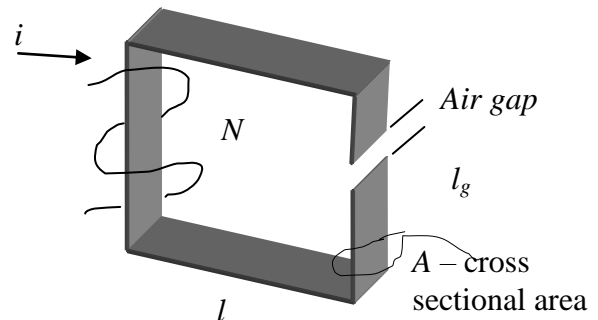


*Frequency:* The first magnetic energy conversion process is between electrical and magnetic. An inductor's magnetic energy is transferred to a capacitor as electrical energy. The energy is cycled back to the inductor causing a frequency. The inductor can be a radio transmitter or an antenna.

*Solenoid:* The second magnetic energy conversion process is magnetic to mechanical. The magnetic field of a single coil can attract a ferromagnetic lever causing mechanical motion. A spring will often return the lever when the magnetic field is released. This device is called a solenoid. If electrical contacts are moved by the solenoid, the device is a relay.

The transfer of energy progresses from electrical to magnetic to mechanical. The reverse process is also applied.

Electrical > magnetic > mechanical

Seldom is there a direct mechanical to electrical conversion. A notable exception is static. Direct electrical to mechanical conversion is most often in the form of heat.
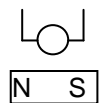


### 5.2.2  Two coils

Machines consist of two coils or inductors. One, the field, provides the magnetic energy for excitation. The second coil, the armature, reacts to the magnetic exciter, converts electrical energy, and provides the load.

The first rotating machines were dynamos, now called generators. The device is driven by mechanical rotation of the shaft. Then, electrical energy is extracted from the terminals. The first motor came about by accident. A dynamo was hooked-up improperly and was found to be a motor.

By definition, a rotating machine is cyclic in nature. Therefore, it is an alternating current device. Note the diagram from the alternating current chapter.
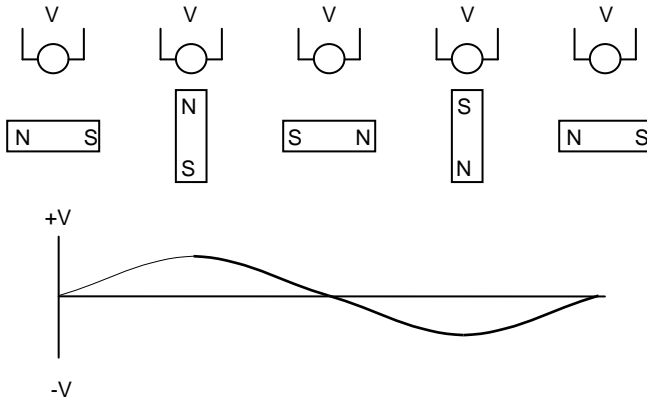
Alternating current is created in a coil of wire by a magnet rotating very close to the wire. As the magnetic pole distance varies, the flux changes, and the magnitude of voltage induced on the coil changes. The voltage is a result of electromotive force. The symbol, E, is frequently used for a voltage source or electromotive force. The number of turns is noted by N and the magnetic flux by $\varphi$.

$$E = N\frac{d\varphi}{dt}$$

Since this is cyclic motion, the change in flux is expressed as a sinusoid operating on the maximum magnetic flux.

$$E = N\varphi\omega\sin\omega t$$

The angular speed, $\omega$, provides the rate per second, while the sinusoidal term varies the magnitude.

The rotating magnet provides the field excitation. The magnet can be permanent or an electromagnet created by an electrically energized coil. The stimulated coil is the armature or load. If electricity is supplied by the armature, the machine is a generator.

The armature can be electrically energized. In this case the coil creates a magnet. The magnet will oppose and attract the poles on the field magnet. This causes the rotor to move. Inertia continues the motion until the next magnetic pole is encountered. The machine is now a motor.

The rotating component of the machine is called a rotor. The stationary component is called a stator. The field can be on either the rotor or stator. Obviously, the armature would be on the other. Making electrical connection to a rotating coil creates a number of issues that a goes to the design of the machine.

The three types of energy in a machine are electric, magnetic, and mechanical. The orientation of the armature is normal to both the magnetic field and to the direction of mechanical motion or torque.

## 5.3 Models

Models are a representation or illustration of a system that allows for investigation of the properties of the system and in some cases prediction of performance. Models are often used in three technical formats. Analysis is looking at how the system works. Design is looking at how to build a system. Application is looking at how to use a system. Therefore, there may be different models of the same system to assist in the various investigations.
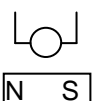
## 5.3.1  Thevenin - Armature

Models are used to electrically represent the machine performance. Machines can be modeled as a Thevenin equivalent voltage and impedance for the armature with a magnetizing field circuit consisting of an inductor with its resistance. The magnetizing inductor induces flux which produces the Thevenin voltage. The model is a two-port network. The Thevenin voltage appears similarly to gain.

There are three fundamental rotating machine models – DC, synchronous, induction – and one fixed machine model – transformer. What is the difference between the machines? It is simply whether steady-state voltage or cyclic voltage is instituted.
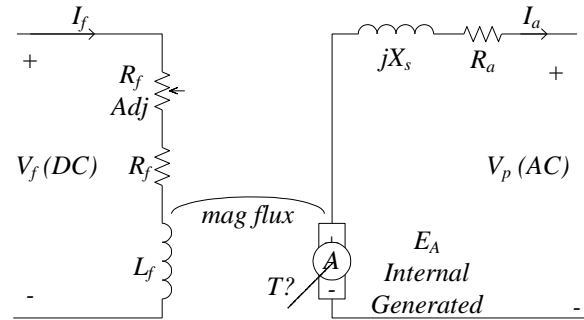
| Rotating Machine | Field V | Armature V | Field Location | Armature Location |
|---|---|---|---|---|
| DC | DC | DC | Stator | Rotor |
| Synchronous | DC | AC | Rotor | Stator |
| Induction | AC | AC | Stator | Rotor |

An induction machine that has both inductors fixed or stationary is called a transformer. Since it is purely electrical and magnetic, without the mechanical rotation, it is much easier to build and typically is the first machine studied.

The input energy and output energy determine the use. A motor has electrical in and mechanical out. A generator has mechanical in and electric out. The same machine can be used in either form. It simply depends on the driver input and the driven output.

| Machine | Input - Driver | Output - Driven |
|---|---|---|
| Generator | Mechanical | Electrical |
| Motor | Electrical | Mechanical |
| Transformer | Electrical | Electrical |

## 5.3.2  Two-port

Circuit elements are modeled as a one-port network with only two terminals or nodes. Most active devices, machines, and networks are modeled as a two-port network. These have two terminals for input and two terminals for output. In some instances, one of the input and output terminals are common.

The diagram shows magnetic coupling between the input and the output. This is merely representative. Any combination of connections and elements are possible. However, the coupling is illustrative of the transfer of energy from the input terminals to the output. Note the same configuration could be used for a transistor or any other network.

In a later chapter, power transmission is investigated by making a two-port network for the line. The components are distributed and are typically measured in Ohms/mile, Ohms/foot, or Ohms/km.

The current into the node and the voltage across the terminals determines the transfer relationships. Two-port models have external measurements which take three forms; impedance, admittance, or hybrid. Impedance models have voltage in terms of current. Admittance models have current in terms of voltage. Hybrid models have both voltage and current as outputs.

If there is a common internal connection, then there are three possible network arrangements. Although the configuration is the same, different names are used in circuit analysis and power systems.
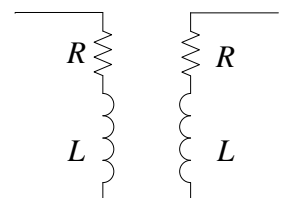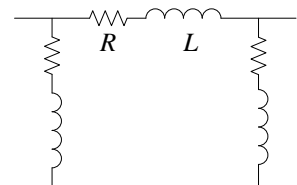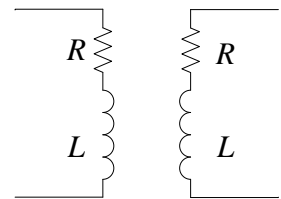
| Circuit | Power |
|---------|-------------|
| T | Y - wye |
| Π | Δ - delta |
| L | L - shunt |

For a three-phase machine, there are three inductors on the armature. These can be connected in only two possible arrangements.
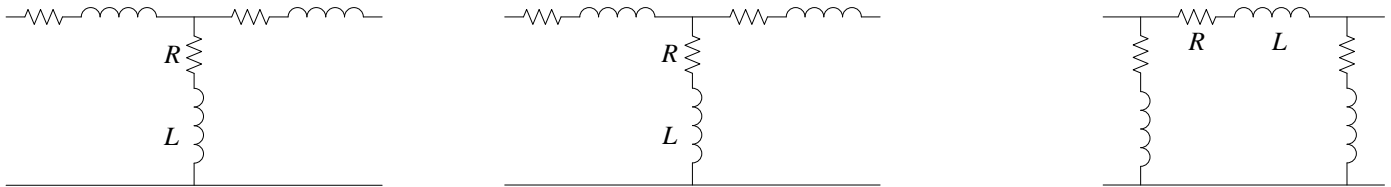


Based on the two port network, models will be developed for each of the four machines. The typical representation is shown. Work the problem on per- phase.

## 5.4  Machine Two-Port Models

Models are simply a representation of a machine. Many texts develop the models through an involved theoretical process. Numerous papers and articles have been written about the design of machines. These are excellent references for more advanced work with motors and generators.

This text approach is to understand concepts and the common relationships between all machines. The principles are oriented to applications engineers. Realizing the field excitation circuit and the Thevenin armature circuit, the model can be simply stated based on a two-port network. The details of each machine are addressed in a separate chapter.
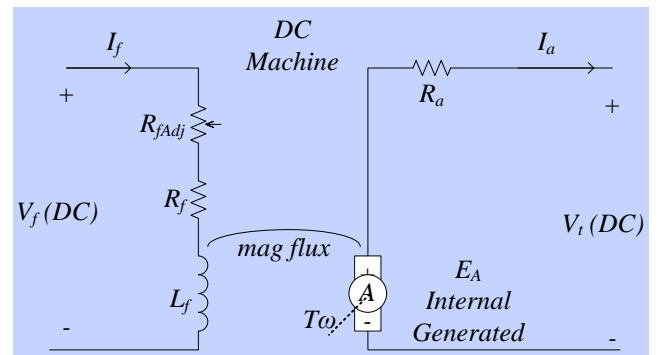
**Common Relationships**

$$\lambda = Li = \varphi \mathcal{R}$$

$$\tau = K\varphi I_f$$

$$E_A = K\varphi\omega = K_1 I_f \omega$$

Use curve of $I_f - vs - E_A$

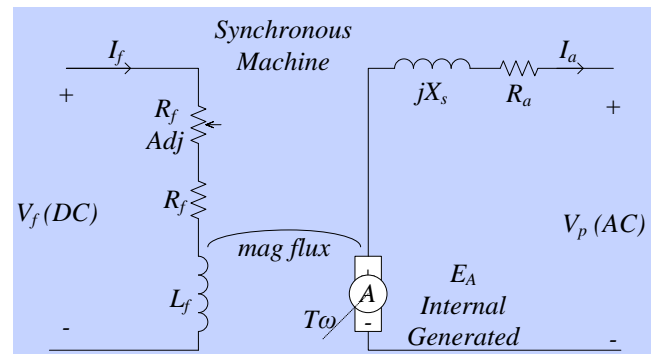$$\frac{E_{A\,actual}}{E_{A\,reference}} = \frac{n_{actual}}{n_{reference}}$$

**DC Machine**

$$V_f = I_f (R_f + R_{fadj})\,\text{DC}$$

$$V_t = E_A - I_A R_A$$

**AC Machine**
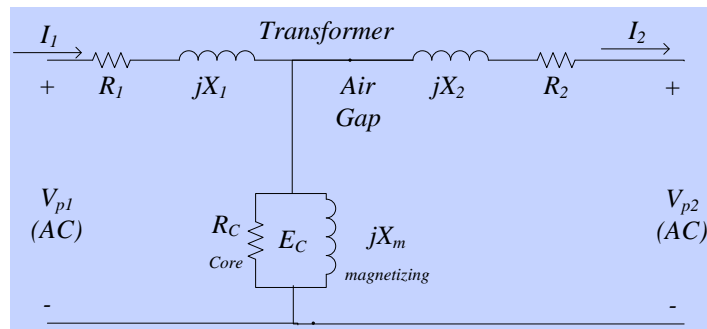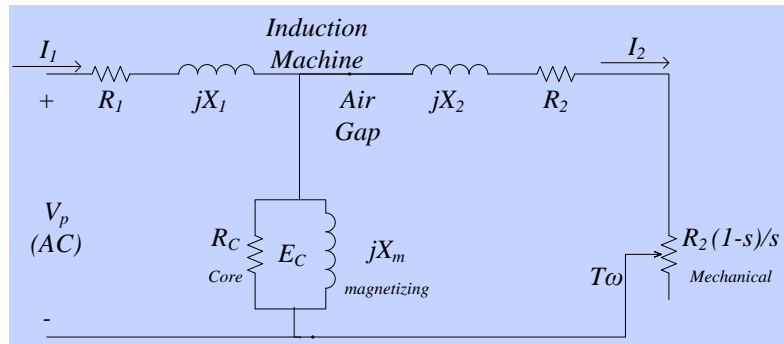
$$V_p = E_A - I_A(R_A + jX_s)$$

**Induction Machine**

$$e = N\varphi\omega\sin\omega t$$

$$E_{max} = N_c\varphi\omega = 2\pi N_c\varphi f$$

$$E_A = \frac{E_{max}}{\sqrt{2}}$$

$$E_\Delta = E_A \qquad E_y = \sqrt{3}E_A$$





# *5.5  Loss Diagrams*

Power is input to machines, converted to another energy form, then output in another form. Each conversion process results in a loss. Losses are represented as a resistance and expressed in terms of power. Loss diagrams graphically illustrate the conversion process.

Electric losses occur in the copper conductor. Magnetic losses arise in the iron core. Mechanical losses include friction and windage as a result of rotation.
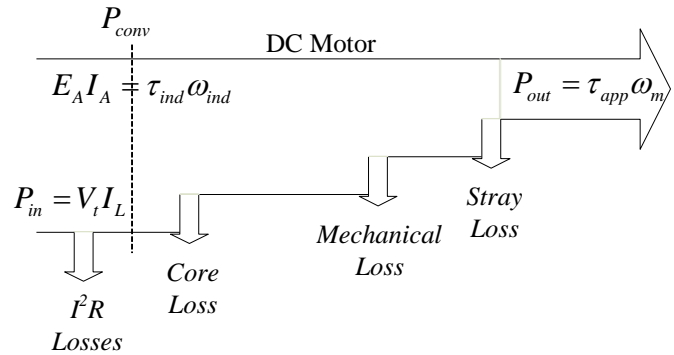


**Losses**

| | |
|---|---|
| $P_{CU} = I^2R$ | electric, copper |
| $P_{FE} = E_C{}^2/R_C$ | magnetic, iron, core |
| $\sim n^3$ | mechanical, friction&windage |
| 2 V | brush |
| 1% | stray |

**Conversion**

| | |
|---|---|
| $P_E = VI$ | electric |
| $P_M = T\omega$ | mechanical |
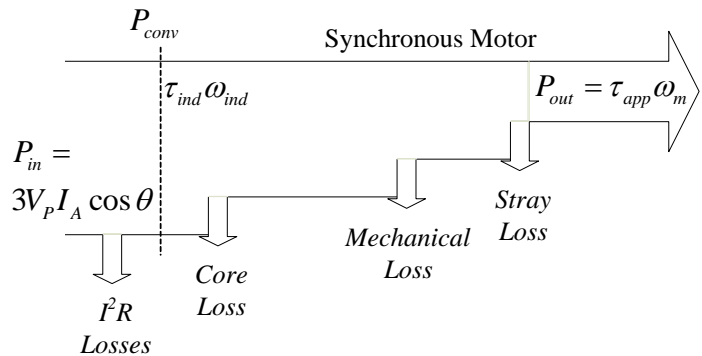
**Efficiency**

$$P_{losses} = P_{copper} + P_{core} + P_{mech} + P_{stray}$$

$$P_{out} = P_{in} - P_{losses}$$
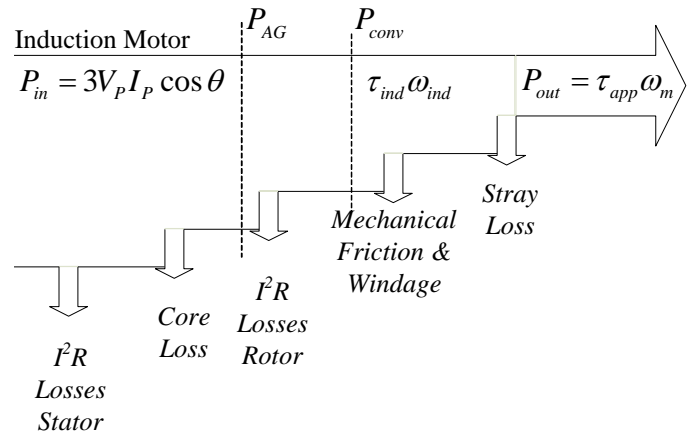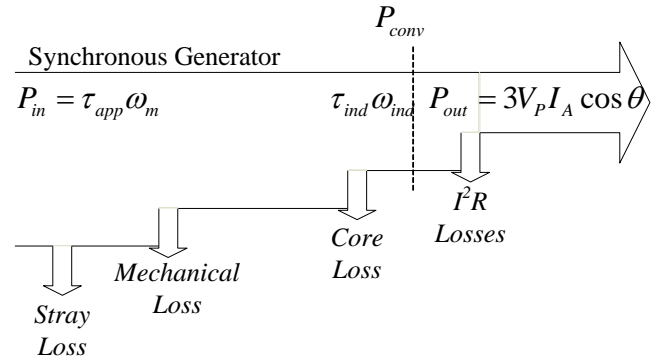
$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{losses}}$$

Power air gap is transferred across the space between the field and armature. Power converted is the changed between mechanical and electrical. Torque induced is the converted power. Torque applied is the power on the shaft and connected to the load.

Synchronous Generator

$$P_{in} = \tau_{app}\omega_m \qquad\qquad \tau_{ind}\omega_{ind} \quad P_{out} = 3V_P I_A \cos\theta$$

$P_{conv}$

$I^2R$ Losses

Core Loss

Mechanical Loss

Stray Loss

Induction Motor

$$P_{in} = 3V_P I_P \cos\theta \qquad \tau_{ind}\omega_{ind} \qquad P_{out} = \tau_{app}\omega_m$$

$P_{AG}$　$P_{conv}$

Stray Loss

Mechanical Friction & Windage

$I^2R$ Losses Rotor

Core Loss

$I^2R$ Losses Stator

## 5.6　Machine Tests

The tests on machines are conducted with the following instrumentation connections. The source is a variable voltage such as dc power supply, ac variac or frequency generator that can adjust the voltage into the machine. The ammeter and the wattmeter may have a shunt that is used to bypass excessive current.

```
(PS) ── Ammeter ──┬── WattMeter ──┬── Machine
                  │               │
               Voltmeter          │
```
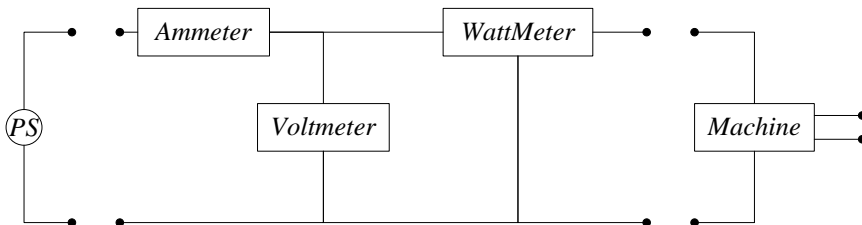
Read voltage, current, and a third parameter at each test.
For open circuit tests, connect as shown with no load. Run at rated voltage.
For short circuit tests, short the terminals of the transformer or block the rotor of the machine. Start at low voltage and increase voltage until near rated current.

| No load / open circuit test | Blocked Rotor / short circuit test |
| --- | --- |
| Set rated voltage, frequency | Reduce V to set I near rated current |
| Read reduced I | Read reduced V |
| I through core / excitation | I through rotor |

The equivalent circuit is expressed as impedance, $Z = R + jX$

## 5.6.1 Third Parameter

Using the Triad Principle, in addition to voltage and current magnitudes, a third parameter is required to determine a circuit. Traditionally power has been preferred for most tests on larger machines. In some circumstances a power meter is not available so an alternative real value, resistance, is easily recorded.

Resistance may be preferred for small loads with current less than 1 A. For a thorough analysis, resistance should be compensated for temperature and ac skin effect. However, at room temperatures with small wires, this is generally not significant.

The calculation process is very similar. With voltage and current and a real parameter, the phase angle can be calculated. From those three parameters, all other items can be determined.

$$\cos\theta = \frac{P}{V \times I} = \frac{R}{V/I}$$

## 5.6.2 Other Relationships

From three measured values, all circuit parameters can be calculated. These can be manipulated in numerous ways. The variety of forms allows considerable creativity in analyzing electrical and magnetic interactions. The trig form is simple.

$$\sin\theta = \frac{Q}{S} = \frac{X}{Z}$$

$$\tan\theta = \frac{Q}{P} = \frac{X}{R}$$

The quadratic form is somewhat more complex.

$$Q = \sqrt{S^2 - P^2} = S\sin\theta = P\tan\theta$$

$$X = \sqrt{Z^2 - R^2} = Z\sin\theta = R\tan\theta$$

The following table contains the items that completely define a circuit. Note the ones that are measured. The remainder are calculated.

| Voltage | Current | Angle | App-Power | Power | Reactive | Impedance | Resistance | Reactance |
|---------|---------|-------|-----------|-------|----------|-----------|------------|-----------|
| V | I | θ | S | P | Q | Z | R | X |
|  |  |  |  |  |  |  |  |  |

# *5.7 Rotating Machines*

Rotating machines can involve numerous different analyses. These include the torque and voltage generation, power output, power losses, and performance.

## 5.7.1 Torque & Voltage Generation

Rotate a coil inside a magnetic field to develop a voltage and torque.

$$e_{ind} = N\frac{d\varphi}{dt}$$

$$E_{induced \atop armature} = K\varphi\omega$$

$$\tau_{induced} = K\varphi I$$

| Symbol | Definition |
|--------|------------|
| φ | Flux |
| p | Number of poles |
| K | Constant of machine design |
| Z | Number of conductors in armature |
| a | Number of circuit paths |
| N | Number of turns |

$$K = \frac{Zp}{2\pi a}$$

Rather than angular speed, the revolutions are often preferred where $n$ = speed in rpm.

$$\omega = 2\pi n$$
$$E_A = K'\varphi n$$
$$K' = \frac{Zp}{60a}$$

AC machines cause flux to change in a sinusoidal fashion.

$$e = N\varphi\omega \sin \omega t$$

Sinusoids are analyzed using effective or RMS values. Although the machines may be three-phase, the analysis operates on a single-phase at a time.

$$E_{max} = N_c\varphi\omega = 2\pi N_c\varphi f$$
$$E_A = \frac{E_{max}}{\sqrt{2}}$$
$$E_\Delta = E_A \qquad E_y = \sqrt{3}E_A$$

The number of magnetic poles and the frequency determine the speed. Frequency, $f$, is measured in Hertz.

$p/2$ repetitions in one rotation

$$f_{elec=} \frac{p}{2} f_{mech}$$
$$f_m = \frac{n_m}{60}$$
$$f_e = \frac{n_m}{60} \frac{p}{2} = \frac{n_m p}{120}$$

## 5.7.2 Power Conversion

Machines are about energy conversion from one form to another. The conversion is generally investigated using power which is energy over time.

$$P = \frac{\Delta W}{\Delta t}$$

Mechanical Power Converted

$$P_{conv} = \tau_{shaft}\omega_m$$

Electrical Power Converted

$$P_{conv} = P_{in} - loss_{stray} - loss_{mechanical} - loss_{core}$$

$$P_{conv} = E_A I_A$$

Output Power

$$P_{out} = P_{conv} - loss_{copper}$$

AC Machine

$$P_{out} = 3V_p I_p \cos\theta = \sqrt{3}V_L I_L \cos\theta$$

DC Machine

$$P_{out} = V_L I_L$$

## 5.7.3  Losses

Losses are encountered anytime there is a resistance or opposition. These are electric or copper, magnetic or ferrous core, and mechanical or friction. The inaccuracies of measurement are compensated as stray losses.

Electric - Copper Losses (I2R)

DC Machine -    $P_A = I_A{}^2 R_A$ (armature)    $P_f = I_f{}^2 R_f$ (field)

AC Machine -    $P_s = 3I_A{}^2 R_A$ (stator)        $P_f = I_f{}^2 R_f$ (field)

Magnetic - Core Losses – Hysteresis & Eddy

$$\frac{E_m}{R_c}$$

Mechanical losses – friction & windage

No load rotational = (mechanical losses + core losses)

~Proportional to $n^3$

Stray losses

miscellaneous - ~1% of output power

Brush losses (DC Only)

$$P_{BD} = V_{BD} I_A \qquad (V_{BD} \approx 2V)$$

## 5.7.4  Performance

Performance is a comparison of maximums and minimums. They are normalized, so they are dimensionless.

Efficiency

$$\eta = \text{efficiency} = \frac{P_{out}}{P_{in}} * 100 = \frac{P_{in} - P_{loss}}{P_{in}} * 100$$

Voltage regulation

$$V_R = \frac{V_{n1-} V_{f1}}{V_{f1}} 100\%$$

Speed regulation

$$S_R = \frac{n_{n1} - n_{f1}}{n_{f1}} \times 100\%$$

Positive $S_R$ means speed drops with load.

## *5.8  Mechanical Power*

The investigation to this point has predominantly looked at the magnetic and electrical performance. Machines are connected to mechanical devices. This section will outline the conversion process for external equipment..

Power is the product of potential across and flow rate (number/time) through the machine

$$S = P*Q = T\omega = FV = vi$$

| | | | |
|---|---|---|---|
| $S$ = pressure | $F$ = Force | $T$ = torque | $v$ = volts |
| $Q$ = vol/time | $V$ = distance/time | $\omega$ = revolutions/time | $i$ = amps (coul/sec) |

A conversion factor ($K$) may arise because of different units in the measured values. A performance factor ($PF$) may be necessary to scale the machine rating.

$$P_{convert} = \frac{P}{K*PF}$$

The conversion factor depends on the units

| | | | | |
|---|---|---|---|---|
| $P_{convert} = Hp$ | $S=psi$ | $Q=gpm$ | $k=1714$ | *1 gpm=5.45 m³/day* |
| $P_{convert} = Hp$ | $S=feet$ | $Q=bpd$ | $k=135663$ | *pu k=56,000* |
| $P_{convert} = Hp$ | $T=ft\text{-}lb$ | $w=rpm$ | $k=5250$ | |
| $P_{convert} = Hp$ | $v=volt$ | $i=amp$ | $k=746$ | |
| $P_{convert} = watt$ | $v=volt$ | $i=amp$ | $k=1$ | |

Efficiency is the ratio of output to input power terms

$$eff = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{(P_{out} + P_{loss})}$$

The efficiency (*eff*) depends on the pump (machine) design

| | | |
|---|---|---|
| triplex=0.9 | duplex=0.85 | centrifugal=0.7 |
| rod=0.9 | beam system=0.4 | motor=0.98-0.94 |

The brake horsepower (BHP) is the required power into the mechanical machine

$$BHP = \frac{S*Q}{K*eff}$$

The speed ratio (SR) determines comparative speeds

$$SR = \frac{motor\ speed}{pump\ speed} = \frac{pump\ sheave\ diameter}{motor\ sheave\ diameter}$$

Load Torque ($T_L$) is the shaft effort required to be input to the load by the motor

$$T_L = \frac{BHP*PF*K}{\omega}$$

The pump performance factor (*PF*) depends on the pump design

| | | |
|---|---|---|
| Duplex – 1.5 | Triplex – 1.31 | Quintuplex – 1.27 |

Starting Torque ($T_S$) is the effort required by the motor during starting

$$T_S = \frac{HP_{rating}*PF*SR*K}{\omega}$$

The motor torque performance factor depends on the motor torque design characteristic.

NEMA B–1.5   NEMA C–2.25  NEMA D–2.75  NEMA E–1.5

Equipment is sized by using the following sequence:

Determine the horsepower of the fluid,

Convert to brake horsepower of the pump

Determine the pump torque
Select a motor with horsepower rating greater than BHP and a NEMA torque design rating greater than the load torque
Size motor and pump sheaves to maintain appropriate pump speed.

⇐⇑⇒
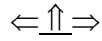
## 5.9  Exemplars

An exemplar is typical or representative of a system. These examples are representative of real world situations.

## 5.10 Applications

Applications are an opportunity to demonstrate familiarity, comfort, and comprehension of the topics.

**Application 4-1**
**SITUATION:**

⇐⇑⇒