Chapter 3 – Magnetic Fields

3.1 Introduction

Circuits are based on the elements constrained to a solid structure. Fields are analogous to a gas where the performance is distributed through space.

3.2 Electric Fields – Electrostatics

3.2.1 Relationships

Electric fields are the result of a charge. When the charge is measured from a point or node it is called statics. When the charge is dispersed in space like a gas the energy is called a dynamic field.

A capacitor is simply two electrically charged conductors that are separated by a dielectric. Energy relationships are used to convert between electric, magnetic, and mechanical systems.

The fundamental relationships for an electric field are listed.

$$W(\text{energy}) = Fs = Vq = NI\varphi \qquad \text{energy conversion}$$

$$s(\text{closed loop}) = 2\pi r$$

$$\mathscr{E}(\text{electric intensity}) = \frac{F}{q} = \frac{V}{s} \quad \text{V/meter}$$

$$\mathscr{D}(\text{electric density}) = q/A = \varepsilon\mathscr{E} \quad \text{Coulomb}/m^2$$

q = charge

1 electron = 1.6021×10^{-19} Coulomb

3.2.2 Permittivity

Permittivity is the dielectric or charge insulation material property.

$$\varepsilon = \varepsilon_r \varepsilon_o$$

$$\frac{1}{\varepsilon_o} = 36\pi \times 10^9 \text{ Farad/m}$$

Force occurs on charge 2 due to charge 1.

$$F = \frac{q_1 q_2}{4\pi\varepsilon r^2}$$

Electric field intensity at point 2 is due to point charge at point 1.

$$\mathscr{E} = \frac{F}{q} = \frac{q_1}{4\pi\varepsilon r^2}$$

A radial electric field comes from a line charge on the z-axis.

$$\mathscr{E} = \frac{\rho_L}{2\pi\varepsilon r}$$

$$\rho_L - \text{line charge density - } \frac{\text{coulomb}}{\text{m}}$$

A plane electric field arises from a sheet charge in x-y plane.

$$\mathscr{E} = \frac{\rho_s}{2\varepsilon}$$

$$\rho_s - \text{sheet charge density - } \frac{\text{coulomb}}{\text{m}^2}$$

Energy is dependent on the electric field, which can be converted to voltage.

$$W = q\mathscr{E}\ s = \frac{1}{2}\varepsilon\mathscr{D}^2$$

$$W = qV = \frac{1}{2}CV^2$$

3.3 Magnetic Fields

3.3.1 Relationships

Magnetic fields are the result of a pole or flux. When the pole strength is measured from a point or node it is called statics. When the magnetics are dispersed in space like a gas the energy is called a dynamic field.

An inductor is simply is simply a coil of wire that creates a magnetic field. Energy relationships are used to convert between electric, magnetic, and mechanical systems.

The fundamental relationships for magnetic devices are listed.

$$W(\text{energy}) = Fs = Vq = NI\varphi$$ energy conversion

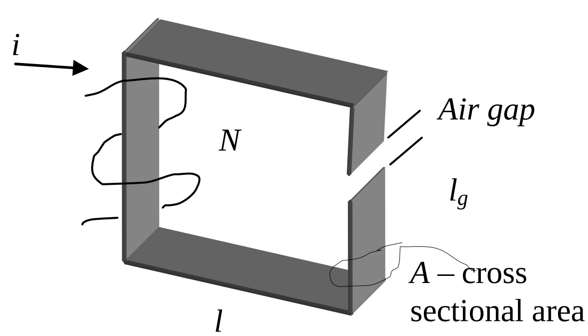
$$s(\text{closed loop}) = 2\pi r$$

$$\mathscr{F}(\text{magneto-motive force}) = NI = \varphi\mathscr{R} = H \bullet dl$$ mmf = Amp-turns

$$\mathscr{H}(\text{magnetic intensity}) = \frac{\text{F}}{\varphi} = \frac{NI}{s}$$ Amp/m

$$\mathscr{B}(\text{field density}) = \varphi/A = \mu H$$ Weber/m^2

$$\mathscr{R}(\text{reluctance}) = \frac{l}{\mu A} = \frac{N^2}{L}$$



3.3.2 Permeability

Permeability is the magnetic property of material.

$$\mu = \mu_r \mu_o$$

$$\mu_o = 4\pi \times 10^{-7}\ \text{Henry/m}$$

$\mu_r(\text{copper}) \approx 2$

$\mu_r(\text{amorphous steel}) \approx 2000 \quad \mu_r(\text{laminated steel}) \approx 6000$

Force occurs on pole 2 due to pole 1.

$$F = \frac{\varphi_1 \varphi_2}{4\pi\mu r^2}$$

$$\lambda(\text{flux linkage}) = n\varphi = LI$$

EXAMPLES

3.1 Given: Laminated iron core

depth of core	.05 m
width of core	.05 m
length of air gap	.0007 m
effective length of core	0.3 m
relative permeability of Fe	2000
air gap flux density	0.5 T
number of turns	200

Find:

a. total flux (include flaring at air gap)

$$B = \frac{\varphi}{A}$$

$$\varphi = BA = 0.5 \times [.05 \times .05] = 0.00125$$

$$\varphi_{AIR} = \varphi \times 1.05 = 0.0013125 \text{ Wb}$$

b. flux density in the iron

$$B_{Fe} = B_{AIR} \times 1.05$$

$$B_{Fe} = 0.5 \times 1.05 = 0.525 \text{ Wb/m}^2$$

c. magnetic intensity in the iron

$$B = \mu H$$

$$H = \frac{B}{\mu} = \frac{B}{\mu_0 \mu_r}$$

$$H = \frac{B}{2000\left(4\pi \times 10^{-7}\right)} = 208.82 \text{ A/m}$$

$$\varphi_{AIR} = \varphi \times 1.05 = 0.0013125 \text{ Wb}$$

d. total mmf required on the core

$$mmf = NI = Hl$$

$$mmf = H \times 0.3 = 62.66 \text{ A-turns}$$

e. current required on the core

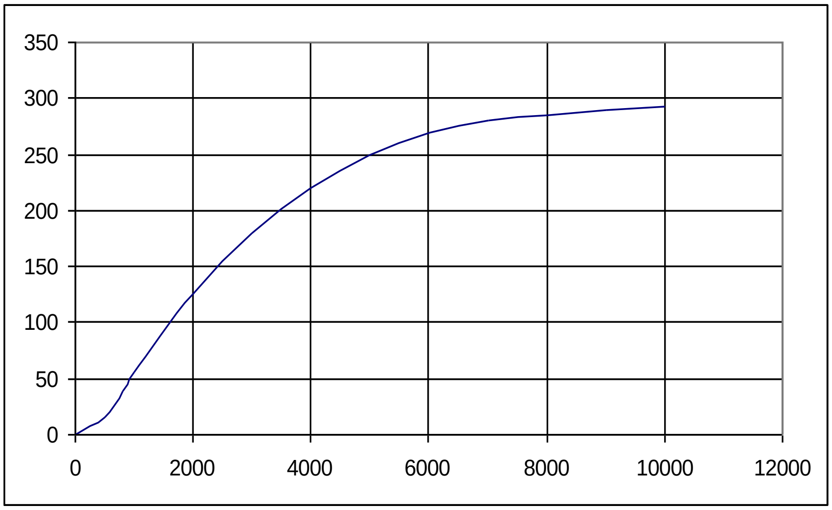
$$mmf = NI$$

$$I = \frac{mmf}{N} = \frac{62.66}{200} \text{ A}$$

3.3.3 Magnetization curve

The magnetic circuit for a machine includes the current and turns, the ferromagnetic metal, and air gaps.

The magnetization curve shown below is a non-linear relationship for the magnetic circuit, and is typical of all magnetic circuits. It is used to show the conversion between representations of magnetic energy.



The first portion of the curve has a physical anomaly near zero. The portion less than 5000 is in the unsaturated region. There is an approximate proportional change in the vertical axis as the horizontal changes. About 5000 is called the knee. That is the transition region. The top portion of the curve above about 5000 is the saturated region. There is very little change in the vertical parameter as the horizontal is increased.

The values are strictly representative. Different material alloys will yield other range of values. Nevertheless, the general shape and form can be used for a variety of problems.

The curve then can represent a number of different relationships. Some of the more common are shown in the table below.

X-axis	**name**	**unit**	**Y-axis**	**name**	**unit**	**Curve**	**name**	**unit**
$\mathcal{F}$	mmf	A-turns	φ	flux	Weber	$\mathcal{R}$	reluctance	
H	intensity	A-turns/m	B	density	Wb/m2	μ	permeability	H/m
I	field I	Amps	V	terminal	Volts		synchronous	
$\mathcal{F}$	field mmf	A-turns	E_a	Internal gen	Volts		dc machine	

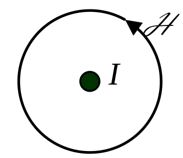
	EXAMPLES
3.1	Given: Magnetization curve shown above Find:
	a. For an intensity H of 5000, what is the density B? Read chart along bottom to 5000. Read vertical to curve. Read result on left column of 250
	b. For a density B of 250 what is the relative permeability?

$$B = \mu H$$

$$\mu = \frac{B}{H} = \frac{250}{5000} = 0.05$$

c. For a flux of 250, what is the mmf?
Read left column to 250
Read right to curve
Read result down to 5000

3. 4 Intensity & Density



The location and direction of a magnetic field are determined by the configuration of the conductor. Note that the intensity and density are related by the permeability μ. Therefore, either can be determined from these equations.

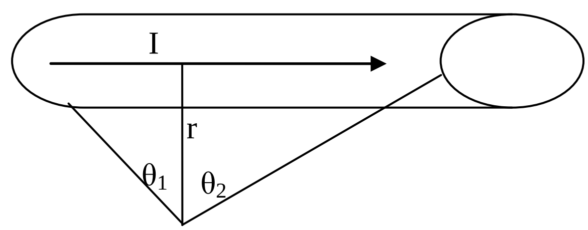
$$\mathcal{B} = \mu H$$

Magnetic flux lines are continuous about a source, and perpendicular at all points to source, in a parallel plane. An arrow shows the direction of a field. When projected on a plane, a 'X' is the tail of the arrow with the field going into the plane. A dot • is the point of the arrow with the field coming out of the page.

The direction is determined by the right hand rule. Curl the right hand with four fingers in the direction of the magnetic field. The thumb points in the direction of the current.

$$\mathcal{H} = \frac{I}{2\pi r}$$

A magnetic field is produced by straight conductor carrying current I.



$$\mathcal{H} = \frac{\mu I}{4\pi r}\left(\sin\theta_1 + \sin\theta_2\right)$$

Energy is dependent on the magnetic field, which can be converted to current.

$$W = \varphi\mathcal{H}s = \frac{1}{2}\mu\mathcal{B}^2$$

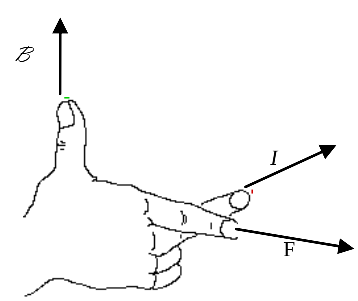
$$W = \varphi I = \frac{1}{2}LI^2$$

Power density in watts per cubic meter is the Poynting vector, the product of the density cross with the intensity.

$$W/V = \mathcal{B} \times \mathcal{H}$$

3.5 Motor & Generator

Motors and generators combine electric, magnetic, and mechanical energy. The three-dimensions for mechanical force, electric current, and magnetic field are related by three fingers of the right hand. The field direction follows the right hand rule.



$$F = i(l \times \mathcal{B})$$

First Finger, x-axis – *F*

Middle Finger, y-axis – I, length, velocity

Thumb, z-axis – $\mathcal{B}$

In another application of the right hand rule, with the fingers curled in the direction of current, the thumb will point in the direction of the magnetic flux, φ.

The motor relationship comes from an electric current flow through a magnetic field. This creates a mechanical force.

$$F = i(l \times \mathcal{B})$$

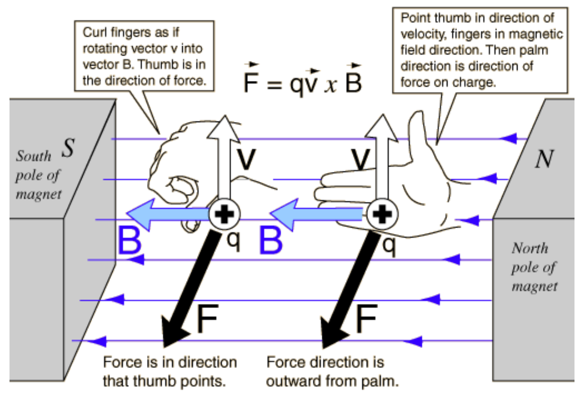
$$F = q(v \times \mathcal{B})$$

The generator relationship comes from a wire moving through a field. This creates an electrical voltage.

$$e(\text{voltage}) = (v \times \mathcal{B}) \bullet l$$

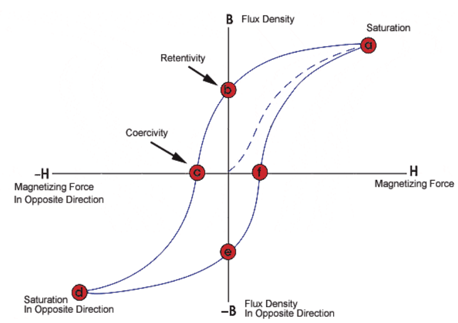
Since voltage and current are not traditionally considered vectors, the length of the line carrying the current is used for the direction.

The cross-product vector directions are determined by the curl of the right hand. Start with the fingers of the hand pointing in the direction of the first vector. With the hand curling from one vector toward the second vector direction, the cross-product result is in the direction of the thumb. The dot product is in the same direction as the two vectors.



3.5.6 Hysteresis and eddy currents

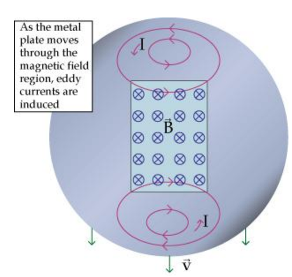
The non-linear nature of magnetic devices creates some rather interesting phenomenon. The non-linear characteristic of a magnet causes it to follow different path when the magnet is being energized and when it is de-energized. This alternative trace is called hysteresis.



Hysteresi**s** is the lag between cause and effect. The system does not instantly respond to the energy applied to it. Rather the system reacts slowly, or does not return completely to its original state. The system's state depends on the immediate history. For example, when pressed putty will assume a new shape, and when the pressure is removed it will not return to its original shape immediately and entirely.

It is hysteresis or residual magnetism that causes data to be stored on a magnetic tape and hard disks. The residual magnetism is also used to get a generator working. In motors, which repeatedly are energized and de-energized, hysteresis causes loss and reduces efficiency.

Another phenomenon of magnetic systems is eddy current. Like the eddy pools in a flowing stream, these are small swirls of current that oppose change in energy. Eddy currents develop on magnetic materials, such as steel, because of anomalies in the molecular structure. To keep the eddy currents from becoming large, the steel core is made in thins layers. Alternatively in some systems, slits may be cut in the steel core.



Eddy currents are used to create dynamic braking. In motors which are continually energized and de-energized, eddy currents causes loss and reduces efficiency.

3.6 Mechanical motion

3.6.1 Relationships

Mechanical motion is the result of mass. When the mass is measured from a point or node it is called statics. When the mass is in motion it is call dynamics. Energy relationships are used to convert between electric, magnetic, and mechanical systems.

The fundamental relationships for three types of field are listed.

$\text{W(Energy)} = Fs = Vq = NI\varphi$	energy conversion
$P = \frac{W}{t} = \tau\omega$	power

3.6.2 Gravity

Gravity constant is the mass property of material.

$$\gamma_o = 8.38 \times 10^{-10}\, N - m^2 / kg^2$$

Force occurs on mass 2 due to mass 1.

$$F = \frac{m_1 m_2}{4\pi\gamma r^2}$$

Energy is dependent on the mass field.

$$F = ma$$

$$W = Fs = mas$$

$W = mgh$ potential energy

$W = \frac{1}{2} mv^2$ average kinetic energy

$W = mv^2$ instantaneous kinetic energy

$W = mc^2$ total energy conversion

3.6.3 Linear v. rotational

The motion of mass has both linear and rotational properties. Because of the rotation of machines, the angular calculations are more common.

Name	Linear	Units		Angular	Units
displacement	s	meter		θ	radian
velocity	v=s/t	m/sec		ω= θ/t	rad/sec
acceleration	a=v/t	m/sec^2		α= ω/t	rad/sec^2
mass	m	kilogram	moment of inertia	J	kg-m^2
force	F=ma	Newton	torque via angular	*T*=J α	Nt-m
			torque via linear	*T*=s x F	Nt-m
energy	W=s·F	Joule		W= *T* θ	Joule
power	P=Fv	Watt		P= *T* ω	Watt

3.7 Mathematical manipulation

Numerous equations and relationships have been broached in this chapter. These are the basis for most electrical, magnetic, and mechanical machine analysis. The study requires manipulation of relationships in a variety of ways and combinations.

There is a simple technique for systematic manipulation: *Begin with a known quantity. Draw a horizontal line so the units can be manipulated until the correct form is obtained. Then the equation will be correct, except for conversion constants.*

As an example, convert revolutions per minute (RPM) to radians per second.

$$\frac{\text{rev}}{\text{min}}\left|\frac{\text{min}}{60\,\text{sec}}\right|\frac{2\pi\ \text{rad}}{\text{rev}} = \frac{\pi}{30}\,\text{rad/sec}$$

3.8 Maxwell

Maxwell's Equations are a summary of all that is electrical and magnetic in a Calculus form. Although they are not easily used, they do provide a mathematically clever summary. The integral form is tedious, but the point form supplies definitions.

$$\nabla \times E = -\partial B / \partial t$$

$$\nabla \times H = J + \partial D / \partial t$$

$$\nabla \bullet D = \rho$$

$$\nabla \bullet E = 0$$

The del is the three-dimensional partial derivative with respect to the x, y, and z axes.

The electric-magnetic energy equation contains all the information in one equation[1]. Note that all the definitions are in the point equation. In addition, all the field information is in the distributed equation. W is energy and V is volume in these equations.

$$W = \phi_z q_y / t$$

$$W = \frac{\phi_z q_y b_{ys} d_t s_y}{t_r V_y}$$

$s =$ axial distance

$b =$ radial distance

$d =$ rotational distance

Realizing the orthogonal nature of the fields, the equations inherently contain the directions and require only vector algebra.

[1] "A Composite Approach to Electrical Engineering," Marcus O. Durham, *Institute of Electrical and Electronic Engineers Region V*, 88CH25617-6/000-143, Colorado Springs, CO, March 1988, pp 143-147.

Applications Engineering Approach to Maxwell and Other Mathematically Intense Problems", Marcus O. Durham, Robert A. Durham, and Karen D. Durham, *Institute of Electrical and Electronics Engineers PCIC*, September 2002.

"Applications Engineers Don't Do Hairy Math", Marcus O. Durham, Robert A. Durham, and Karen D. Durham, *Proceedings of 35th Annual Frontiers in Power Conference*, OSU, Stillwater, OK, October 2002.

"Electromagnetics in One Equation Without Maxwell", Marcus O. Durham, *American Association for Advancement of Science - SWARM*, Tulsa, OK, April 2003.

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3.9 Exemplars

An exemplar is typical or representative of a system. These examples are representative of real world situations.

3.10 Applications

Applications are an opportunity to demonstrate familiarity, comfort, and comprehension of the topics.

Application 5-1
SITUATION:

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