Chapter 4 – AC

4.1 Introduction

Signals and waveforms are the driving source for networks and circuits. The most common is the sinusoid. Because of mathematical relationships, any waveform can be represented as the sum of multiple sine and cosine waves. Therefore, the description of sinusoidal performance is an integral part of any circuits analysis.

The time domain signal representation contains all the components of signal responses including DC, transient, and sinusoidal.

$$y(t) = F + (I - F)e^{-t/\tau}\cos(\omega t + \theta)$$

Fortunately most analysis can be greatly simplified. Phasor transforms are used with only magnitude and phase shift to completely express the sinusoidal signal levels.

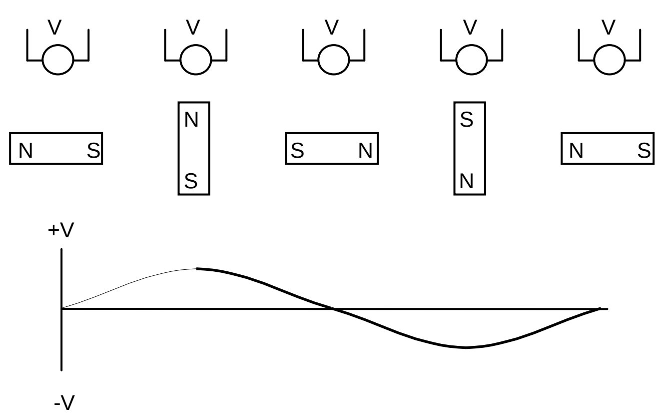
The vast majority of power is delivered and consumed using only the sinusoidal response. Cyclic or alternating current (AC) will be illustrated with applications. AC is explained in terms of voltage, current, and angle which are used to calculate the ratio, impedance, and the produce, power.

Single-phase power requires only two wires to feed the components of the system. Three-phase power requires three current carrying conductors.

4.2 Alternating current

Alternating current is created in a coil of wire by a magnet rotating very close to the wire. As the magnetic pole distance varies, the magnitude of voltage induced on the coil changes.

The chart illustrates the magnet at four positions with the fifth position the same as the starting point.



Consider the magnet starts horizontal, in the distance farthest from the coil. The voltage will be zero. As the magnet rotates clockwise, the voltage will increase until the magnet is nearest the coil. That will be the maximum voltage. Then the magnet will rotate away from the coil, with the voltage decreasing. When the magnet is close again, but with opposite polarity, the voltage will again be at maximum, but with a negative polarity. The rotation continues until the magnet is at the starting point.

A similar result is obtained by a coil of wire rotating in a magnetic field.

The curve illustrates the 360° rotation of the magnet and the resulting cycle for the voltage. For machines in the Western Hemisphere, the electrical frequency of rotation is 60 times per second or 60 Hertz (Hz). So each cycle is 1/60 of a second. The time that the voltage crosses the axis can then be expressed equally well as degrees or seconds.

2π radians = 360 degrees = 1 cycle = 1 revolution.
$\theta = 2\pi f t = \omega t$
$t = \theta / 2\pi f = \theta / \omega$

4.3 Phasor Transform

Steady state AC analysis is used when the source is operating at a constant frequency. Because of inductors and capacitors, an angle change or phase shift is created between voltage and current associated with the impedance.

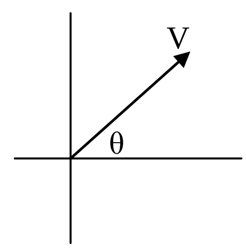
Frequency (*f*) is related to the time it takes for a waveform to repeat. It is the number of repetitions or cycles in a second (Hz).

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

where

ω = angular frequency in radians per second

On many power systems the frequency is fixed or constant for normal, or steady state operations. Long before Gene Roddenberry created *Star Trek* and the phasor, they were in use to describe the performance of electrical systems. The *phasor transform*, ***P***, can be applied to provide a concise phasor notation at the fixed frequency.



$\boldsymbol{P}[V_{peak} \cos(\omega t + \theta)] = V_{RMS}\angle\theta$

$$V_{peak} = V_{RMS}\sqrt{2}$$

where

θ = phase shift between voltage and current cause by L and C.

Because of the phase shift, complex numbers are often used to manipulate the mathematics. Complex numbers involve both a real and imaginary part. The imaginary part is defined as multiplied by "*j*" which is referred to as the imaginary unit.

$$j = \sqrt{-1} = e^{j\frac{\pi}{2}};$$

$$\frac{1}{j} = -j = e^{-j\frac{\pi}{2}}$$

Euler's formula allows conversion of rectangular coordinates to polar. It gives a representation of the angle impact on the sinusoids.

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

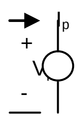
The time varying voltage (AC) is inverse of the phasor transform magnitude and angles described above.

$$v = R_e(\sqrt{2}V_{RMS}\, e^{j\theta}e^{j\omega t})$$

$$v = \sqrt{2}\,V_{RMS}\,\cos(\omega t + \theta)$$

4.4 Single-phase

A machine contains mechanical rotation, magnetic poles, and electric circuit. A single-phase machine contains only one component set of these three physical components. A single-phase component, whether source or load, can be represented by a two-node network. The voltage across the component is the phase voltage, V_P, and the current through is the phase current, I_P.

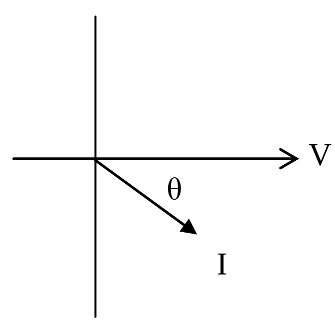


The voltage has a time associated with its magnitude, as does the current. This time is represented as an angle which is a portion of a complete rotational cycle.

From the measured voltage, current, and time, three things can be calculated. Impedance is the ratio of the voltage to current. Power is the product of the voltage and current. Time delay is the difference in the time when the voltage and current are at a minimum.

Impedance ______________________

The current is related to the voltage by Ohm's Law. By convention, voltage is used as the reference for measurement comparison. Bold letters represent a vector with magnitude and angular direction. Normal letters are scalar values of magnitude only.



Z = V / I

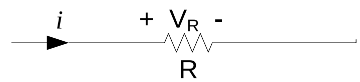
$$Z\,\underline{/\,\theta} = \frac{V_P\,\underline{/\,0^o}}{I_P\,\underline{/{-\theta}}}$$

Current lags the voltage by the angle of the impedance. If the impedance has a negative angle value, obviously the current angle will be positive and lead the voltage.

The impedance is the phasor combination of the three physical elements in an electrical circuit. Resistance, R, is a characteristic of the conductor material that opposes the flow of current. An inductance, L, arises if the conductor is wrapped into a coil. A capacitor, C, arises when two conductors are adjacent.

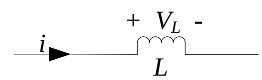
Resistance is independent of time or the resulting phase angle.

Z = R



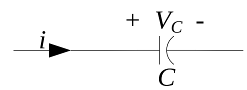
However, an inductor operates as a reactance at an angle of 90°. In a rectangular system this is the +j direction.

Z = +j X_L
= +j ω L

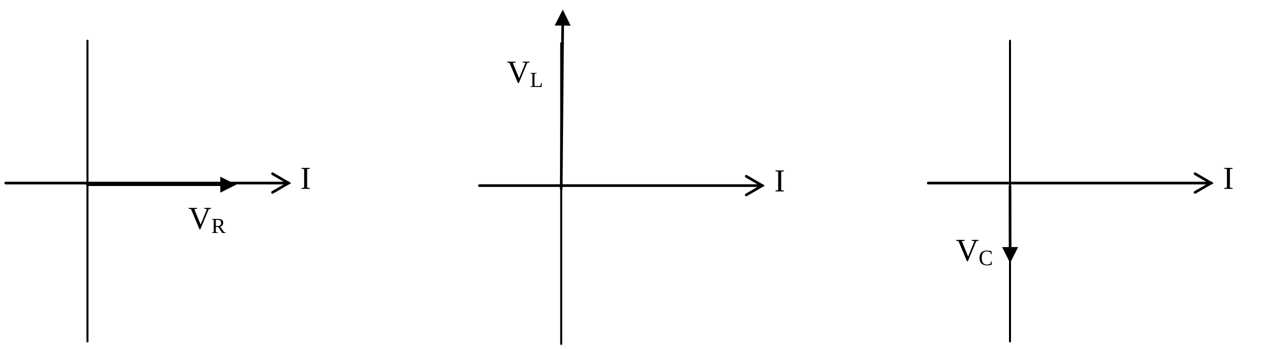


Conversely, a capacitor operates as a reactance at an angle of -90°. In a rectangular system this is the -j direction.

Z = -j X_C
= -j / ω C

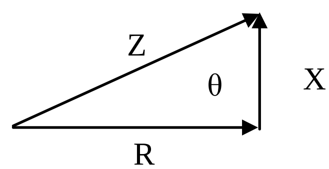


The phasor diagram for the resistor, inductor, and capacitor are respectively illustrated.

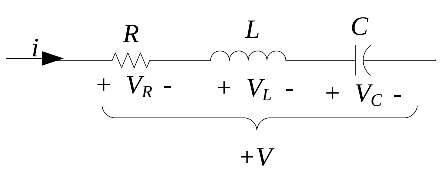


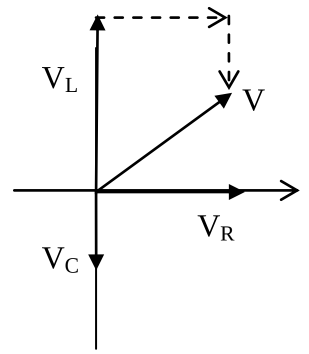
Obviously, an inductor and a capacitor are complementary devices that can be used to balance a system. The combination of the impedance elements results in a value with an angle.

$Z \underline{/\theta} = R + j\,X_L - j\,X_C$
$= R + j(X_L - X_C)$



The combination of resistor, inductor, and capacitor can be shown in a phasor of voltages. In a series circuits, the current will be the same in all elements, so the impedance is proportional to the voltage. In a parallel circuit the voltage is the same across all elements, so the impedance is proportional to the current.

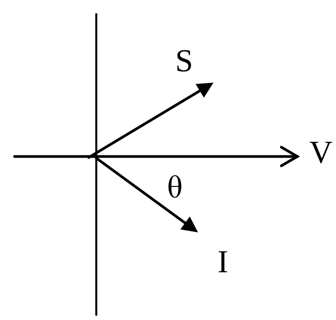




Apparent power ___________________

Apparent power is the product of voltage and current conjugate. The conjugate is simply changing the sign of the current.

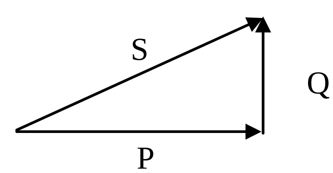
$$\mathbf{S = V\,I^*}$$
$$= (V_P \angle 0^o)\,(I_P \angle -\theta)^*$$
$$= V_P\, I_P \angle \theta$$



The angle has numerous interesting relationships. The current vector is the conjugate of the impedance and power angle. This angle represents the time delay between the current and the voltage crossing the zero-axis.

Because of the angle associated with the apparent power, the power can be separated into two components, the real power, P, and the reactive power, Q. The real power is a mechanical conversion of the resistance. The reactive power represents the magnetic energy stored in the inductor and the electrical energy stored in the capacitor.

$$S \angle \theta = P + j\,Q_L - j\,Q_C$$
$$= S\,(\cos\theta + j\sin\theta)$$



The components of apparent power can also be represented in terms of the angle associated with the current and impedance.

$$P = S\cos\theta = V\,I\cos\theta$$
$$Q = S\sin\theta = V\,I\sin\theta$$

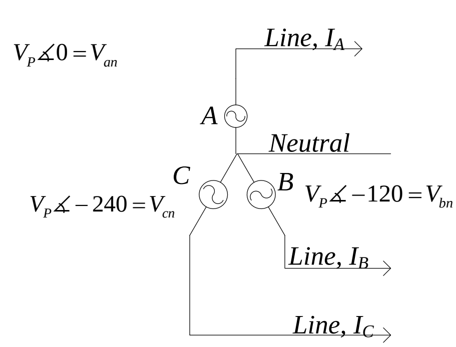
In the power world the real component factor is called the power factor. It is the ratio of the real mechanical component to the total. Another common factor is the ratio of the reactive component to the real mechanical component.

$$pf = \cos\theta = P / S = R / Z$$
$$\tan\theta = Q / P = X / R$$

Three-phase

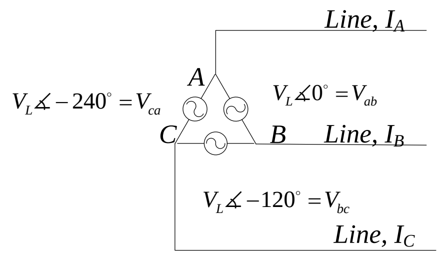
Three-phase indicates 3 sources, 3 lines, and 3 loads. In essence, three-phase is three single-phase systems connected together. Based on the original illustration, the magnets are placed on the rotating member called rotor, separated by 120°. Similarly, the coils are placed on the stationary member called stator, separated by 120°.

The three-phase system can be represented as a three-node network. The arrangement of three coils with six wires can be yields two possible connections to obtain three-phase. If a common connection is made between all three components, then the three phases are connected to the remaining wire on each component. This is called a wye configuration. The connections to the lines are labeled as A-B-C.



If a connection is made between only two components, then there are only three possible connections. These connect to the three phase lines. The arrangement is called a delta configuration.

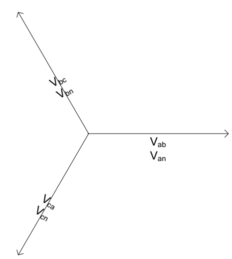
The voltage across the phase component, V_P, and the current through is the phase current, I_P are the same as the single-phase values. However the terminal voltages are different and depend on the configuration. When balanced, a single relationship for the phase values can be related.



Wye ___________________________

In the wye configuration, each phase is connected to a common terminal called the neutral. Then the voltage on each phase is equal to the phase magnitude separated by an angle of 120°. By convention, the A phase is assumed to be at an angle of zero degrees.

V$_{AN}$ = V_P $\underline{/\ 0^o}$
V$_{BN}$ = V_P $\underline{/\text{-}120^o}$
V$_{CN}$ = V_P $\underline{/\text{-}240^o}$



In voltage designations, the first subscript is assumed to be the higher or positive voltage compared to the second subscript.

The terminal voltage is the voltage between the lines, V_{LL}. The respective line to line voltages are V_{AB}, V_{BC}, and V_{CA}. The magnitude and angle can be calculated from the phase voltages.

The relationship between the lines and phases are simple vector calculations. The reference voltage starts with an angle of zero. The line voltage is the difference in the corresponding phase voltage.

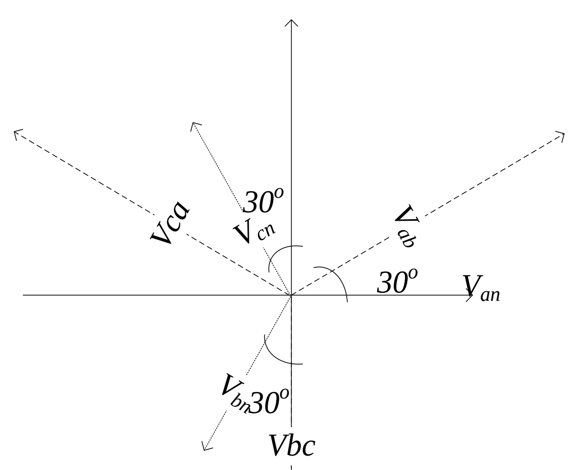
$$
\begin{aligned}
V_{ab} &= V_{an} - V_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ \\
&= V_P(1 + j0) - V_p(-0.5 - j\,\sqrt{3}/2) = V_P(1.5 + j\,\sqrt{3}/2) \\
&= \sqrt{3}V_p\angle 30^\circ
\end{aligned}
$$

The other two line to line voltages can be determined similarly. Alternately, they can be found by recognizing the shift of 120° between phases.

$$V_{bc} = \sqrt{3}V_p\angle 30^\circ - 120^\circ$$

$$V_{ca} = \sqrt{3}V_p\angle 30^\circ - 240^\circ$$

The relationships are visually shown in a phasor diagram.



The current in each line is simply the current in the phase.

I$_A$ = I_P $\underline{/\text{-}\theta}$
I$_B$ = I_P $\underline{/\text{-}\theta\ \text{-}\ 120^o}$
I$_C$ = I_P $\underline{/\text{-}\theta\ \text{-}\ 240^o}$

The current in the neutral is the sum of the current in the three phases.

$$\mathbf{I}_N = \mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C$$

In a balanced system, the magnitude of the currents is the same. When the angles are resolved, it is found that they cancel each other.

$$I_N = 0$$

A summary of the line and phase relationships is based on the AN phase.

$$\mathbf{V}_{LL} = \sqrt{3}\, V_P \, \underline{/\, 30^o}$$
$$\mathbf{I}_L = I_P \, \underline{/\text{-}\theta}$$

Often only magnitudes are expressed since the mechanical shift of 30^o does not change, regardless of the circuit or calculations. Likewise, the impedance angle is often excluded since it is the same in all phases and is not readily measured with most current meters. These common simplifications give magnitudes but do not express phase shifts which impact power.

$$\boxed{V_{LL} = \sqrt{3}\, V_P}$$

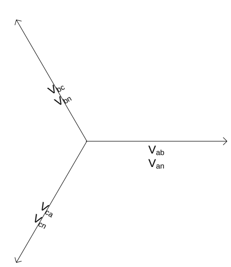
$$\boxed{I_L = I_P}$$

Delta

In the delta configuration, each phase component is connected to an adjacent phase. Then, the terminal voltage is equal to the phase voltage. Again because of the geometry of the generator, the voltage on each phase is equal to the phase magnitude separated by an angle of 120^o.

Since the terminal of a delta system can be connected to a wye system, it is important that the phase and subscript convention be consistent.

$$\mathbf{V}_{AB} = V_P \, \underline{/\, 0^o}$$
$$\mathbf{V}_{BC} = V_P \, \underline{/\text{-}120^o}$$
$$\mathbf{V}_{CA} = V_P \, \underline{/\text{-}240^o}$$



The sum of the line current and phase currents at each terminal is zero. Therefore, the line current is the difference in the current in the phases that are connected to the terminal.

$$\begin{aligned}\mathbf{I}_A &= \mathbf{I}_{AB} - \mathbf{I}_{CA} \\ &= I_P \, \underline{/\text{-}\theta} - I_P \, \underline{/\text{-}\theta - 240^o} \\ &= \sqrt{3}\, I_P \, \underline{/\text{-}\theta\ \text{-}30^o}\end{aligned}$$

The other two currents can be determined similarly. Alternately, they can be found by recognizing the shift of 120° between phases.

$\mathbf{I}_B = \sqrt{3}\, I_P\, \underline{/-\theta\ -30^o} - \underline{/-120^o} = \sqrt{3}\, I_P\, \underline{/-\theta\ -150^o}$
$\mathbf{I}_C = \sqrt{3}\, I_P\, \underline{/-\theta\ -30^o} - \underline{/-240^o} = \sqrt{3}\, I_P\, \underline{/-\theta\ -270^o}$

The sum of the current in the three phases is zero since there is no neutral conductor..

$\mathbf{I}_N = \mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0$

A summary of the line and phase relationships is based on the A line.

$\mathbf{V}_{LL} = V_P\, \underline{/\ 0^o}$
$\mathbf{I}_L = \sqrt{3}\, I_P\, \underline{/-\theta\ -30^o}$

In most references the angles are not included similar to the wye configuration discussion.

$$\boxed{V_{LL} = V_P}$$

$$\boxed{I_L = \sqrt{3}\, I_P}$$

From these considerations, the following relationships hold, whether delta or wye.

$$I_N = I_A + I_B + I_C$$

$$V_{LN} = \frac{V_{LL}}{\sqrt{3}} \angle -30^\circ$$

$$V_P = \frac{V_L}{\sqrt{3}} \angle -30^\circ$$

The significance of the equations is the phase always lags behind the line..

Phase 30° behind (lags) line

V_{an} is 30° behind V_{ab}, and 90° ahead of V_{bc}

Phase sequence

A visual representation is convenient for identifying the sequence between phases. The graph also illustrates the angular correspondence between phases and lines. The curves can be either voltage or current, since they are related by the angle, θ. Voltage is the reference. Therefore, it is more frequently drawn.

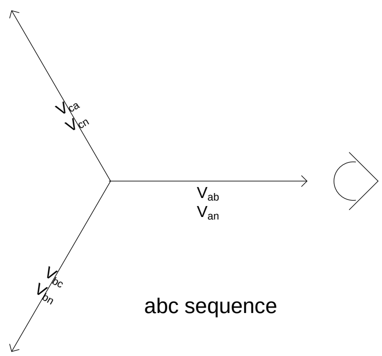
A three-phase machine will rotate based on the sequence or order of the terminal connections. If any two of the terminal lines are exchanged, the direction of rotation will be reversed. Normal rotation is called an ‘ABC’ sequence. If the rotation is changed the negative sequence is called ‘CBA’

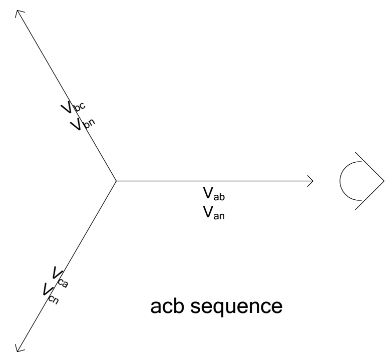
Phase sequence is used to determine the direction of rotation of machines and compatibility for connection to other circuits.

1) "Look" in the zero axis
2) Then rotate phase relationships in a counter-clockwise (CCW) direction
3) Note the sequence of the phasors that cross the zero-axis as they come by.

   For example looking at a terminal connection, the phasors may be in the order AB – BC – CA.
4) Determine the sequence by taking only the first letter in each pairing.

   In the example, A-B-C. That is a positive sequence.





- a~~b~~,b~~c~~,c~~a~~ → abc
- a~~n~~,b~~n~~,c~~n~~ → abc

- a~~b~~,c~~a~~,b~~c~~ → acb
- a~~n~~,c~~n~~,b~~n~~ → acb

Three-phase power _______________

Three-phase impedance has no real meaning. Impedance is the ratio between the voltage and current in each phase. Therefore, it is not converted to terminal or line values.

On the other hand, three phase power has several representations. In the most basic definition, it is the sum of the power in all three phases. In a balanced system the power in each phase is equal in magnitude and separate by the mechanical configuration of 120^{O}. In a complete three-phase system, the mechanical angles will cancel, but the phase shift between voltage and current persists.

$$\begin{aligned} \mathbf{S}_{3\varphi} &= \mathbf{S}_{P1} + \mathbf{S}_{P2} + \mathbf{S}_{P3} \\ &= V_{P1}\, I_{P1}^{*} + V_{P2}\, I_{P2}^{*} + V_{P3}\, I_{P3}^{*} \\ &= 3\, V_P\, I_P \underline{/\ \theta} \end{aligned}$$

Phase values are the internal parameters of a machine. As such, they are not easily measured. The terminal or line values are a more common representation. The relationship between the terminal values and the phase values depends on the wye or delta configuration. However, because of the symmetry, the three-phase power will be the same for both wye and delta.

Consider a wye configuration. A similar analysis of a delta design would yield the same results. The angles associated with the mechanical phase shifts are not included. As has been discussed, these will cancel each other.

$$V_{LL} = \sqrt{3}\, V_P$$
$$V_P = 1/\sqrt{3}\, V_{LL}$$

$\mathbf{I}_L = I_P \underline{/-\theta}$
$\mathbf{I}_P = I_L \underline{/\ \theta}$

$\mathbf{S}_{3\varphi} = 3\ \mathbf{S}_{1\varphi}$
$= 3\ [1/\surd 3\ V_{LL}\ \ I_L \underline{/\ \theta}\ ]$
$= \surd 3\ V_{LL}\ I_L \underline{/\ \theta}$

The three-phase power can be segregated into real and reactive components exactly as the single-phase variables.

$S_{3\varphi} \underline{/\theta} = P + j\ Q_L - j\ Q_C$
$= S\ (\cos\theta + j \sin\theta)$

The components of apparent power can also be represented in terms of the angle associated with the current and impedance.

$P_{3\varphi} = S_{3\varphi} \cos\theta$
$= 3\ V_P\ I_P \cos\theta$
$= \surd 3\ V_{LL}\ I_L \cos\theta$

$Q_{3\varphi} = S_{3\varphi} \sin\theta$
$= 3\ V_P\ I_P \sin\theta$
$= \surd 3\ V_{LL}\ I_L \sin\theta$

To recap, the three phase power is expressed in terms of phase voltage and current or line voltage and current.

$\mathbf{S}_{3\varphi} = 3\ V_P\ I_P \underline{/\ \theta}$
$= \surd 3\ V_{LL}\ I_L \underline{/\ \theta}$

Recap

The relationships between phase and line values are identified for three-phase systems. The terms are separated into magnitude and directions.

For a three-phase system, there are only two possible connections, wye and delta.

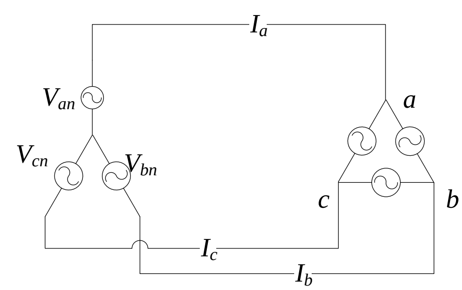
The magnitude of the voltage and current changes by a factor of √3.

Wye	Delta
$\sqrt{3}V_P = V_L$	$V_P = V_L$
$I_P = I_L$	$\sqrt{3}I_P = I_L$

V_L leads V_P by an angle of $\underline{/\ 30^o}$

I_L lags I_P by an angle of $\underline{/-30^o}$

Impedance angle is $\underline{/\ \theta}$



Complex apparent power can be determined using phase or line values.

$$1\varphi\ S = V_P I_P$$

$$3\varphi\ \mathrm{S} = 3V_P I_P^*$$ easier for wye

$$3\varphi\ \mathrm{S} = \sqrt{3} V_L I_L^*$$ delta or wye

The complex value can be expanded to magnitude and angle. Apparent power contains both real and reactive components. Real is dependent on resistance and reactive is dependent on inductors and capacitors.

Power factor = $\cos\theta = \dfrac{P}{S}$

$$P = |S| \cos\theta$$

$$Q = |S| \cos\theta$$

$$|S| = \sqrt{(P^2 + Q^2)}$$

Impedance is only a single phase relationship. There is no relationship to line values.

$$Z = \frac{V_P}{I_P}$$

Neutral current is the sum of the current in the phases.

$$I_N = I_A + I_B + I_C$$ for wye w/ neutral

$$\mathrm{I_N} = 0$$ for wye w/o neutral or delta

A key principle of three phase power affirms that regardless of whether the current is Y or Δ, the line currents lag the line-to-neutral voltages by the phase (impedance) angle.

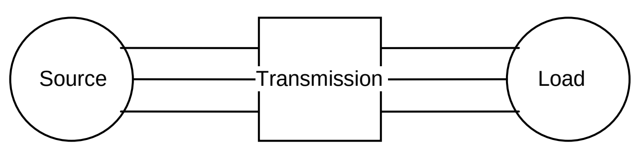
$$Z = R + jX$$

$$|Z| = \sqrt{(R^2 + X^2)}$$

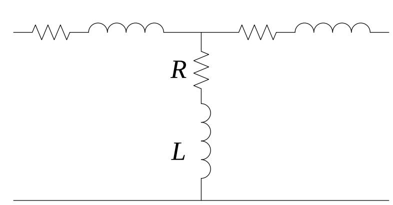
$$\theta = \tan^{-1} \frac{X}{R}$$

One-line diagram

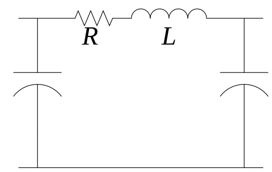
A power system consists of sources or generators, transmission lines, and loads which are commonly motors.



In a single-phase system, the source and load are each represented as a two-node network. However, for a three-phase system, each is a three-node network. The network can be either delta or wye. The three-nodes can also be analyzed as a two-port network. In network analysis a delta is also called a pi-network, while a wye is also called a T-network.

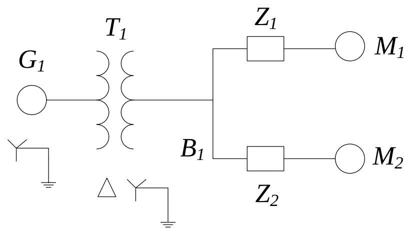


Each transmission line is a combination of resistors and inductors in series and capacitors in shunt. Therefore, each line is also a three-port net. These are connected in such a way that the input is a three-port with the output as a three-port and a transfer function between them that is impedance.



Traditional circuit analysis of three-phase networks would require many lines and connections. This actually makes the investigation appear quite complicated.

As a result, most power analysis uses a one-line diagram. The sources or generators (G), loads or motors (M), and transformers (T) are identified with a note about wye or delta connection. Transmission lines (Z) are described by an impedance. A connection is called a bus (B). Voltage, current, and power transfer is calculated at each bus.



The description and specification of each component is displayed on the one-line. If there is inadequate room, a table may be used. Impedance is often in terms of Ohms per distance. The voltage angle is typically zero.

Location/ condition	V_{LL}	/V	I_L	/I	Pf or /Z	S	P	Q	Z	R	X	distance
G1												
T1												
B1												
Z1												
M1												
Z2												
M2												

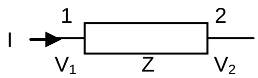
The known values are placed in the table. Other values in the table are calculated as required based on the known parameters. The voltage, current, and power transfer is simply calculated at each node.

One-phase, one-port transfer _______

One-phase is simply a one-port network. It is represented by an equivalent impedance and may include a source. A generator is often modeled as a voltage source in series with an inductive reactance. A transformer is a resistor and inductive reactance. A transmission line is an impedance. A motor may be an impedance.

In looking at the one-line diagram, it is apparent that the voltage and current on one side of an impedance can be related to the voltage and current on the other side. The relationships can be determined by the power transfer across the impedance.

Consider a two node model for impedance with the input at terminal 1 and the output at terminal 2.



The voltage across the impedance network is the difference in the voltage on the input and output.

$$\mathbf{V}_{12} = \mathbf{V}_1 - \mathbf{V}_2$$

The current through the impedance is related by Ohm's Law.

$$\mathbf{V}_{12} = \mathbf{I}\ \mathbf{Z}$$

The apparent power is the product of the voltage and current.

$$\begin{aligned} \mathbf{S} &= \mathbf{V}\ \mathbf{I}^* \\ &= \mathbf{V}_{12}\ \mathrm{I}\ \underline{/\theta} \end{aligned}$$

Ohm's Law and power can be combined to have different terms for power.

$$\begin{aligned} \mathbf{S} &= \mathbf{V}_{12}\ \mathbf{I}^* \\ &= \mathrm{I}^*\mathrm{I}\ \mathrm{Z} = \mathrm{I}^2\ \mathrm{Z} \\ &= \mathrm{V}_{12}^{\ 2} / \mathrm{Z} \end{aligned}$$

The impedance has been defined as the sum of the resistance, the inductive reactance and the capacitive reactance.

$$\mathbf{Z} = \mathrm{R} + \mathrm{j}\ \mathrm{X}_L - \mathrm{j}\ \mathrm{X}_C$$

In many problems, one of the impedance elements will dominate. Then the others can be reasonably ignored.

Power transfer

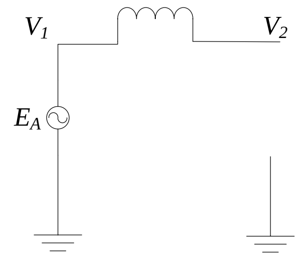
Two special cases of impedance are considered. The first is a resistance only. The second is reactance only.

The power through a resistor is real power only. It represents conversion to real mechanical energy. Losses are in the form of heat. There is no angle or phase shift associated with a resistance. The relationships for real and reactive power are determined when $\theta = 0$.

$$\begin{aligned} \mathrm{P} &= \mathrm{S} \cos\theta \\ &= \mathrm{V}_{12}\ \mathrm{I} \\ &= \mathrm{I}^2\ \mathrm{R} \\ &= \mathrm{V}_{12}^{\ 2} / \mathrm{R} \end{aligned}$$

$$\mathrm{Q} = \mathrm{S} \sin\theta = 0$$

The second example has a reactance only. Power can be *transferred* across the reactance but there is no power *converted*. Reactive power can also be transferred. The power transfer causes a phase shift in the voltage, denoted by the angle δ. These relationships will be stated simply rather than derived.



The standard power computation is the starting relationships.

$$S = VI^*$$

$$P = S\ pf$$

The real and reactive power transfer contains the phase shift.

$$P_{1\to 2} = \frac{V_1 V_2 \sin(\delta_1 - \delta_2)}{X}$$

$$Q_{1\to 2} = \frac{1}{X}\left[|V_1|^2 - |V_1||V_2|\cos(\delta_1 - \delta_2) \right]$$

To create a consistent analysis, assume the angle relationships for voltage and apparent power.

$$V = |V|\angle 0$$

$$\angle S = \angle Z$$

From this, the remaining angles and the current are found.

$$\angle I = \angle S$$

$$pf = \cos\theta$$

$$I = |I|(\cos\theta + j\sin\theta)$$

Although the illustration has been single-phase, it can readily be extended to three phase using the relationships developed earlier. An application of the power transfer is the transfer from Bus 1 to Motor 2 in the one-line diagram. This is also used with generators which have only a reactance.

Angle and power factor

The three items that are necessary to be measured are voltage, current, and phase angle between them. The angle is a representation of time. Although time is easily measured, angle is difficult. Therefore, it is represented from other terms.

One common term is power factor. It is the ratio of real to apparent values. It can be expressed either from impedance or power. The ratio is also the cosine of the phase angle.

pf = R/Z = P / S = cos θ

Note the duality of the relationships.

pf = R/(V/I) = P / V*I) = cos θ

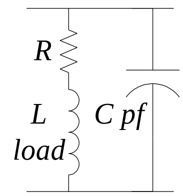
Another ratio commonly used to express characteristics of transformers and motors is X/R ratio. It is easy to see that this term is closely related. It is the tangent of the phase angle.

$X / R = \tan\theta$

Most loads on power systems consist of transformers and motors. These inductive devices create a large reactance. As a result, the power factor is significantly less than one. Since the current is lagging the voltage by the impedance angle, the power factor is called lagging.

Power factor can be connected by two methods. A synchronous machine can have the field current adjusted to create either a leading or lagging power factor. Therefore, it can be used to correct for lagging power factor from other machines. Machines used to correct power factor are called synchronous condensers. Since synchronous machines are used primarily on very large systems, this is not a common technique.

A more common technique to compensate for the inductive lagging is to add capacitors. To correct the power factor to nearer unity, capacitance is added in parallel to the load. At unity, the system would be at resonance and the capacitive reactance would equal the inductive reactance.



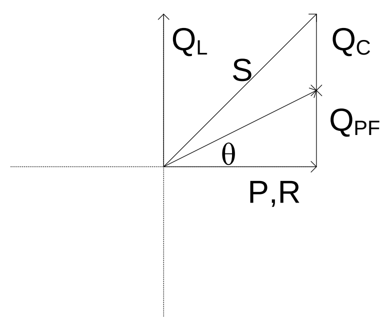
$$X_L = X_C$$

Power factor should not be corrected past unity, since power factor would begin decreasing again, but the current would be leading the voltage.

To determine the capacitance necessary for a particular power factor, find the reactance or the reactive power. Then subtract the existing reactance. The real component, power and resistance, is unchanged.

Example:

Find the capacitor necessary to correct power factor from 0.8 to 0.95 for a load of 100 kW.



$$\frac{P}{S} = \frac{R}{Z} = \cos\theta = pf$$

$$\frac{Q}{S} = \sin\theta$$

$$Q = \frac{P}{pf}\sin\left(\cos^{-1} pf\right)$$

Calculate the reactive power for the inductor and after power factor correction. The difference is the reactive power in the capacitor

$$Q_L = \frac{100}{0.8}\sin\left(\cos^{-1} 0.8\right) = 75$$

$$Q_{PF} = \frac{100}{0.9}\sin\left(\cos^{-1} 0.9\right) = 48.4$$

$$Q_C = Q_L - Q_{PF} = 26.6 \text{ KVAR}$$

The capacitor value is for a single phase voltage. If three-phase, each capacitor would be one-third of the total capacitive reactance.

$$Q = \frac{V^2}{X_C} = \frac{V^2}{1/2\pi fC}$$

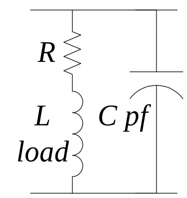
$$C = \frac{Q}{2\pi fV^2}$$

Notice the striking similarity between unity power factor correction and a radio frequency transmitter. They are exactly the same. An inductor is in parallel with a capacitor. This creates an oscillator at the frequency where the reactances are equal. Hence, at unity power factor the system would oscillate.

$$X_L = X_C$$

$$j\omega L = \frac{1}{j\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$



Summary

These calculations determine the external performance of electrical equipment.

Use per unit calculations if multiple voltages are used in the problem (i.e. transformers).

Use the following equations to obtain the desired quantity.

Complex apparent power – volt-amps, VA

$$S = VI^* = P + jQ = |S|(\cos\theta + j\sin\theta)$$

Real Power – watts, W

$$P = VI\ pf = S\ pf = vi$$

resistance only, Q=0

Reactive Power – volt-amp reactive, VAR

$$Q = VI\sin(\cos^{-1} pf) = VI\sin\theta$$

Q > 0 for inductive load, lagging pf

Q < 0 for capacitive load, leading pf

Power Factor

$$pf = \cos\theta = \frac{P}{S} = \frac{vi}{VI^*}$$

pf=1 when $Z = R$ or $jX_L = -jX_C$

X/R

$$\tan\theta = \frac{X_L}{R}$$

Convert Hp to real power (KW).

$$P = \frac{?\,Hp}{} * \frac{0.746kW}{Hp} * \frac{1}{eff}$$

Alternately, convert Hp to apparent power.

$$S = \frac{?\,Hp}{} * \frac{0.746kW}{Hp} * \frac{1}{eff} * \frac{1}{pf}$$

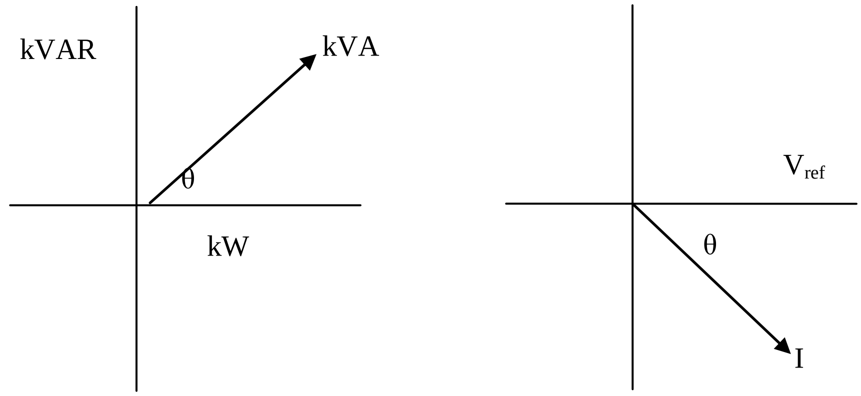
The angle has several relationships. It is also the time delay between the voltage and current crossing the axis (going through zero value). The conversion is

2π radians = 360 degrees = 1 cycle

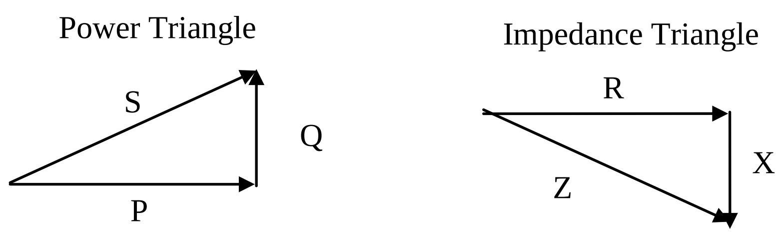
$$\theta = \angle V - \angle I = \angle Z = \angle S = \cos^{-1} pf = 2\pi ft$$

$$t = \frac{\theta}{2\pi f}$$

The relationship between the power terms is shown graphically below.



The relationship of the power terms is often illustrated with complex triangles.



Impedance is the ratio of voltage and current (Ohm's Law)

$$Z = \frac{V}{I} = R + jX = \sqrt{(R^2 + X^2)}$$

Complex Numbers are easiest to manipulate in the following manner:

Add & Subtract P's and Q's
Multiply & Divide: Convert P & Q to S & ∠, multiply (divide) magnitudes, add (subtract) angles

Phasor rotation is used to explain the relationship between the lines of a three phase power system

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4.9 Exemplars

An exemplar is typical or representative of a system. These examples are representative of real world situations.

Exemplar 4-1
SITUATION:

A paper mill is supplied by a 13.8kV, 3-phase, 60 Hz system with appropriate transformers.

The total load is as follows:

Induction Motors	600Hp Efficiency – 85% power factor – 0.8 lagging
Heating and Lighting	100kW unity power factor
Synchronous Motors	200Hp efficiency – 90% leading power factor

The synchronous motors are being operated at rated kVA and are over excited to corrent the plant power factor to 0.95 lagging.

It is desired to increase the mill output by 20%

A plant survey indicates that the installed induction motor capacity is adequate for this increase, but the synchronous machines are at rated kVA.

It is suggested that it might be possible to increase the output of the synchronous motors by a sufficient amount by reducing the excitation to unity power factor and providing power factor correction with a static bank of capacitors.

REQUIREMENTS:

a) Determine if it is possible to increase the output of the synchronous motors by 20% by reducing excitation without exceeding kVA ratings. Explain your answer

b) Determine the capacitance per phase, delta connected, to correct the power factor to 0.95 lagging if the capacitors are connected across the 13.8kV line.

SOLUTION:

Given Existing System:

Induction motors:

$$P = \frac{600Hp}{} * \frac{0.746kW}{1Hp} * \frac{1}{85\%(eff)} = 526.6kW$$

$$S = \frac{526.6kW}{0.8(pf)} = 658.2kVA$$

$$Q = \sqrt{658.2^2 - 526.6^2} = 394.9kVAR$$

Heat & Light:

$$P = 100kW$$

$$S = \frac{100kW}{1.0(pf)} = 100kVA$$

$$Q = \sqrt{100^2 - 100^2} = 0kVAR$$

	S	P	Q	Pf
Induction Motors	658.2	526.6	394.9	0.8
Heat & Light	100	100	0	1.0
Synchronous Motors		165.8		
Plant		792.4		

Synchronous Motors:

$$P = \frac{200Hp}{} * \frac{0.746kW}{1Hp} * \frac{1}{90\%(eff)} = 165.8kW$$

a) Plant:

$$P = 792.4kW$$

$$S = \frac{792.4kW}{0.95(pf)} = 834.1kVA$$

$$Q = \sqrt{834.1^2 - 792.4^2} = 260.5kVAR$$

	S	P	Q	Pf
Induction Motors	658.2	526.6	394.9	0.8
Heat & Light	100	100	0	1.0
Synchronous Motors		165.8		
Plant	834.1	792.4	260.5	0.95

Synchronous Motor:

$$Q = Q_{plant} - Q_{ind}$$

$$= 260.5 - 394.9 = -134.4kVAR$$

$$S = 165.8 - j134.4 = 213.4\measuredangle -39.09$$

$$pf = \cos(-39.09) = -0.776$$

	S	P	Q	Pf
Induction Motors	658.2	526.6	394.9	0.8
Heat & Light	100	100	0	1.0
Synchronous Motors	213.4	165.8	-134.4	-0.78
Plant	834.1	792.4	260.5	0.95

By reducing exciting of synchronous motors to give unity power factor, P of synchronous motors increases to 213.4kW.

$$\%\text{ increase} = \frac{(213.4 - 165.8)}{165.8} = 28.7\%$$

So, synchronous load increased by more than 20%, planned upgrades are possible.

	S	P	Q	Pf
Induction Motors	789.9	631.9	473.9	0.8
Heat & Light	100	100	0	1.0
Synchronous Motors	213.4	199	-77	0.93
Plant	1012.0	930.9	396.9	0.92

b) With 20% plant increase (20% increase in induction motor load and 20% increase in synchronous motor load)

$$\text{Q needed for 95\% pf} = 930.9\tan(\cos^{-1}0.95) = 930.9\tan(18.2^\circ) = 306.1kVAR$$

$$Q_{cap} = Q_{total} - Q_{95\%pf} = 396.9 - 306.1 = 90.8kVAR$$

$$Q_{cap/ph} = \frac{Q_{cap}}{3} = \frac{90.8kVAR}{3} = 30.3kVAR$$

$$S = \frac{V^2}{Z} \qquad Q = \frac{V^2}{X_c}$$

$$C_{ph} = \frac{Q_{cap/ph}}{V^2\omega} = \frac{30,300}{13,800^2 * (2\pi 60)} = 0.422\mu F/ph$$

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4.10 Applications

Applications are an opportunity to demonstrate familiarity, comfort, and comprehension of the topics.

Application 4-1

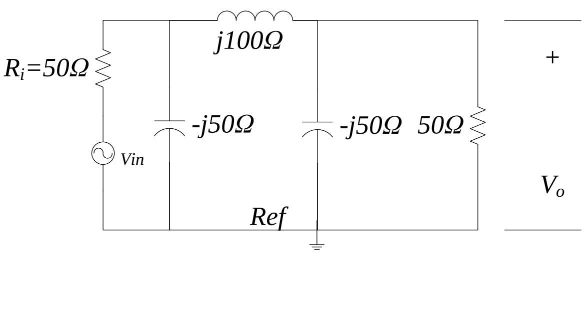
SITUATION:

The circuit shown in the figure below is the pi representation of a transmission line.

REQUIREMENTS:

Write a set of nodal equations for the circuit the “Ref” as the reference or common terminal.

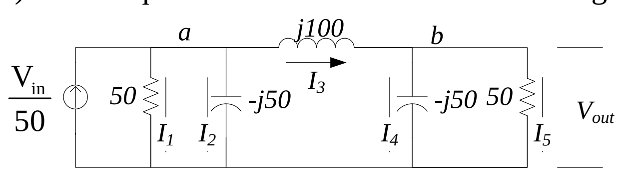
Solve the equations of requirement (a) above for Vo if $V_{in} = 100\measuredangle 0°$



SOLUTION:

Simple circuits problem

a) Nodal equations use current – convert voltage sources to current sources.



$$I_1 = \frac{v_a - 0}{50} \qquad I_2 = \frac{v_a - 0}{-j50} \qquad I_3 = \frac{v_a - v_b}{j100} \qquad I_4 = \frac{v_b - 0}{-j50} \qquad I_5 = \frac{v_b - 0}{50}$$

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