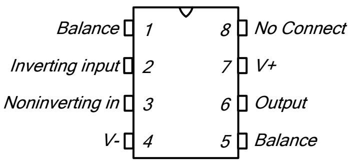
Chapter 6 – Active Device – Op Amp

6.1 Introduction

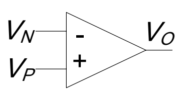
Active devices are energy amplifiers. Passive devices are simply modeled as RLC components. The operational amplifier (OpAmp) is the most common active device. The device is powered by direct current (dc) but is often used with cyclic signals.

6.2 Ideal Operational Amplifiers



An ideal op-amp has infinite input impedance, zero output impedance, and almost infinite gain. Open loop mode is operating without any external devices. The open loop voltage gain, *A* is described by the relationship.

$$V_0 = A(V_P - V_N)$$

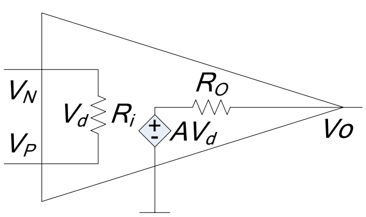


A is large (>10^4) and V_+ and V_- is small enough that it does not saturate the amplifier. For the ideal operational amplifier, assume that the input currents are zero and that the open loop gain *A* is infinite. Then, when operating linearly, the voltage difference is $V_N - V_I = 0$.

The ideal op amp has the following characteristics

1. Infinite open-loop gain, A ≈ ∞.
2. Infinite input resistance, $R_I \approx \infty$.
3. Zero output resistance, $R_O \approx 0$.

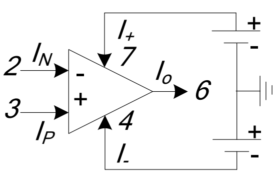
6.3 Non-ideal equivalent circuit



No op-amp meets the ideal characteristics. The non-ideal op amp has an input resistor. The output is from a voltage controlled source driven by the input voltage. The network can be inserted in a traditional circuit for analysis.

As a non-ideal device, the analysis must also include the power supply. All current into the device must be accounted.

$$I_0 = I_P + I_N + I_+ + I_-$$



The LM741/NE741/uA741 op-amps are the most popular. Although this is a variation of the early device, the present models are frequency compensated device. The bi-polar types are low-noise and replacing the older- op-amps.

6.4 Difference Amplifier

Two ungrounded input terminals create a difference amplifier. The device amplifies the voltage difference between the inputs.

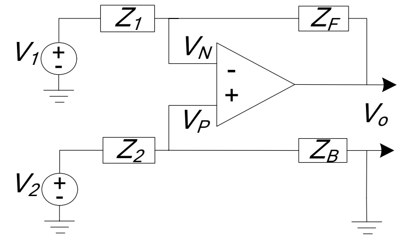
The general form is shown. The significant relationships are calculated.

$$A = \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_P - V_N}$$

The op amp has a high Z input. That implies the voltage between the terminals is essentially 0. So, there is no current flow.

$$V_P - V_N = 0$$

Compare the current on the negative and positive input branches. Use those functions to calculate the output voltage.



$$I_I = \frac{V_1 - V_N}{Z_1} = \frac{V_N - V_0}{Z_F}$$

$$I_N = \frac{V_2 - V_P}{Z_2} = \frac{V_P - 0}{Z_B}$$

The circuit is a voltage divider between the feedback side and balancing side.

$$\boxed{V_0 = -V_1 \frac{Z_F}{Z_1} + V_2 \frac{Z_B}{Z_1} \frac{Z_1 + Z_F}{Z_2 + Z_B}}$$

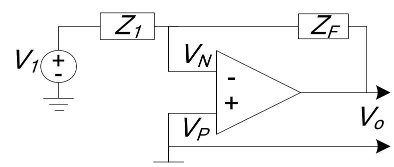
This particular relationship completely encompasses all possible combinations and can be referred to as a generic op amp. However, most circuits are much less complex as shown below.

A difference amplifier can compare two signals or voltages. The output will switch states when there is a difference between the inputs.

6.5 Inverting Amplifier

An inverting amplifier is the basic circuit for an op amp. A Thevenin source is connected to the inverting terminal. This is a signal in series with an impedance. A feedback impedance is connected from the output back to the inverting input. The non-inverting input is grounded. The output is measured relative to ground.

Apply KCL at the inverting node.



$$I_P = I_N$$

$$\frac{V_1 - V_N}{Z_1} = \frac{V_N - V_0}{Z_F}$$

The inverting amp has the inverting input grounded.

$$V_P = V_N = 0$$

Calculate the closed loop gain.

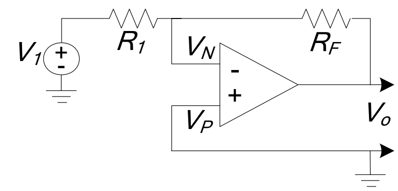
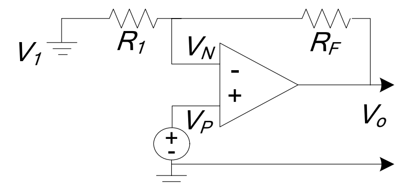
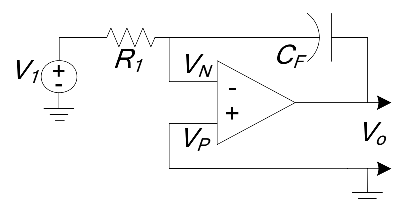
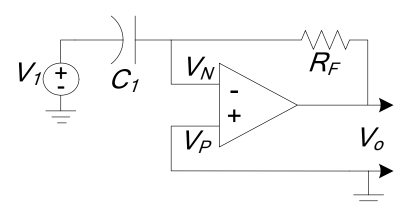
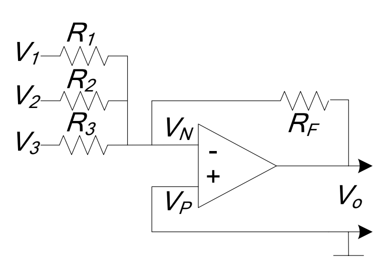
$$\frac{V_1}{Z_1} = \frac{-V_0}{Z_F}$$

$$A = \frac{V_0}{V_1} = \frac{-Z_F}{Z_1}$$

This form can be used to calculate all other connections and uses of the op amp. The table below shows the different applications closed loop gain and diagram. The calculations will not be developed.

6.6 Op Amp Math Device

The connection of the op-amp changes the gain. The closed loop gain is the ratio of the feedback impedance to the input impedance. Complex reactance creates mathematics functions of integrator and differentiator.

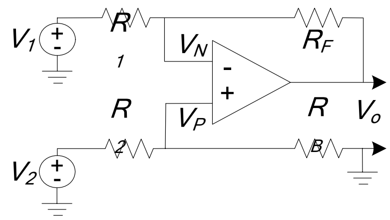
Op Amp Application	Schematic
Inverting Amplifier Voltage on – terminal $$\frac{V_0}{V_{IN}} = A = \frac{-Z_F}{Z_1} = \frac{-R_F}{R_1}$$	
Non-Inverting Amplifier Voltage on + terminal $$\frac{V_0}{V_{IN}} = A = \frac{R_1 + R_F}{R_1}$$	
Integrator Capacitor in feedback $$\frac{V_0}{V_{IN}} = A = \frac{-Z_F}{Z_1}$$ $$= \left(\frac{1}{R_1}\right)\left(\frac{-1}{sC}\right) = -\frac{1}{sCR_1}$$	
Differentiator Capacitor on input $$\frac{V_0}{V_{IN}} = A = \frac{-Z_F}{Z_1}$$ $$= (-R_F)(sC) = -sCR_F$$	
Summer Multiple sources on same input $$V_0 = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$$	

Subtractor

Difference between two inputs & rejects common.

$$\frac{R_1}{R_F} = \frac{R_2}{R_B}$$

$$V_0 = \frac{R_F}{R_1}(V_2 - V_1)$$



EXAMPLES

Ex 1.3-1

Situation: Inverting amplifier with R1 = 10K, A = 100

What is: Rf?

$$A = \frac{-Z_f}{Z_1}$$

$$Z_f = -AZ_1 = -100(10K) = -1M\Omega$$

The minus shows it is inverting

Ex 1.3-2

Situation: R1 = 100K, V1=5V, Cf=1pf

What is: output voltage?

$$A = \frac{V_0}{V_I} = \frac{-Z_f}{Z_1} = -sCR_f$$

$$V_0 = -V_I \frac{1}{sC_f R_1} = \frac{-5}{s(1\times10^{-6})10^5} = \frac{-50}{s}$$

$$v = -50$$

Ex 1.3-2

Situation: The differential amplifier has all impedance elements equal to 1,000 Ohms.

What is: Vo

$$V_0 = -V_1 \frac{Z_F}{Z_1} + V_2 \frac{Z_B}{Z_1} \frac{Z_1 + Z_F}{Z_2 + Z_B}$$

$$V_0 = -V_1 \frac{1K}{1K} + V_2 \frac{1K}{1K} \frac{1K + 1K}{1K + 1K} = V_2 - V_1$$

Why is: Vo equal to the difference in the inputs?

There are no losses in the op amp, so the output is equal to the input.

Ex 1.3-2

Situation: Non-ideal op amp with Open loop gain = 1000, Ri=1 MΩ, Ro=1Ω, Vin=3 V peak to peak, and Rload =8Ω.

What is: Vo

$$V_0 = 1000V_d \frac{8}{8+1}$$

$$= 888.8(3) = 2667$$

6.7 Application - DAC

A digital to analog converter (DAC) can be constructed using the summer circuit. The input voltage represents a bit. It has a value of only 0 or 1. The zero represents the off state of 0 volts. The one is scaled by the power supply level. A 5-volt supply would the 5 x 1 = 5.

The most significant bit (MSB) of the DAC would be voltage source 1. The least significant bit (LSB) would be the last source.

The feedback and first input resistance should have the same value. Greater than 10K Ohm is desired to reduce loading on the input signal. The second bit resistor should be 2(R_1). Each successive bit should be a power of two greater. M represents the bit number multiplier.

$$R_m = 2^{m-1} R_1$$

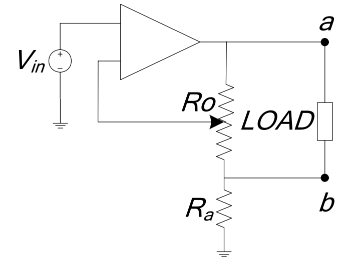
$$R_f = R_1$$

The number of bits determines the resolution of the output. A one-bit would have values of 0 or 1. A two bit DAC has 0 - .5 – 1.

	EXAMPLES
Ex 1.3-3	Situation: A 3-bit DAC has R1 = 10K. What is: Rf, R2, R3? $R_f = R_1 = 10K$ $R_m = 2^{m-1} R_1$ $R_2 = 2^1(10K) = 20K$ $R_3 = 2^2(10K) = 40K$
Ex 1.3-4	Situation: DAC input is [101] What is: $V_0 = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right)$ $-V_0 = \frac{10K}{10K}(1) + \frac{10K}{20K}(0) + \frac{10K}{40K}(1)$ $-V_0 = 1 + 0 + .25 = 1.25$
	Situation: System voltage is 5V What is: scaled output voltage? $-V_0 = DAC(V_S) = 1.25(5) = 6.25V$ The voltage exceeds the source because of the gain in the op amp.

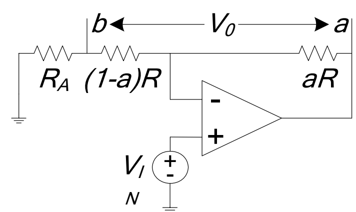
Chapter 6 Problems

SITUATION:
The operational amplifier circuit shown below is connected to provide a variable output resistance at terminals a-b.



REQUIREMENTS:
Find a Thevenin equivalent circuit as a function of the potentiometer setting α at terminals a-b. Assume the op-amp to be ideal and that R is much larger than R_A.

SOLUTION:
Circuit is a non-inverting amplifier (voltage on + terminal). Redraw into standard form.



Find Thevenin Equivalent voltage (V_{TH}).
Open circuit terminals, leave sources active, calculate voltage across open terminals

$$V_{TH} = v_{ab}$$

$$v_{ab} = I_A(\alpha R + (1-\alpha)R)$$

$$= I_A R$$

$$\therefore I_A = {v_{ab}}/{R}$$

$$V_{in} = I_A(R(1-\alpha) + R_A)$$

$$= \frac{v_{ab}}{R}(R(1-\alpha) + R_A)$$

$$v_{ab} = \frac{V_{in}}{R}(R(1-\alpha) + R_A)$$

Find Thevenin Equivalent Impedance (Z_{TH})
Short voltage sources, open current sources, then calculate series/parallel resistances. Alternately, short circuit the terminals, leave sources active, then find I_{SC}.

$$Z_{TH} = \frac{V_{TH}}{I_{SC}}$$

$$I_{SC} = \frac{V_{in}}{R_A}$$

$$R_{TH} = {V_{TH}}/{I_{SC}} = \frac{\left({RV_{in}}/{(R(1-\alpha) + R_A)} \right)}{{V_{in}}/{R_A}}$$

$$= \frac{RR_A}{(R(1-\alpha) + R_A)}$$

Calculate gain.

$$\frac{V_{out}}{V_{in}} = A_v = \frac{\frac{V_{in}}{R}(R(1-\alpha) + R_A)}{V_{in}} = \frac{R(1-\alpha) + R_A}{R}$$