Chapter 4 – Equivalent Circuits

4.1 Introducion

$$S = VI^*$$

$$Z = \frac{V}{I}$$

$$S(j\omega) = P + j(Q_L - Q_C)$$

$$Z(j\omega) = R + j(X_L - X_C)$$

$
universal engineering symbol

$, t, quality
engineering trade-offs

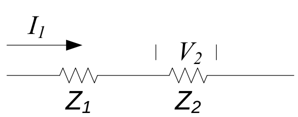
Equivalent circuits are used as a method to solve particular problems without going through the complete Kirchhoff Current Law (KCL) or Voltage Law (KVL process. These processes can be derived one-time, then applied as circumstances arise. Equivalent elements are impedances and sources.

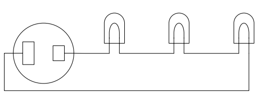
1. Equivalent impedances are combinations of KVL and KCL to reduce the problem to simpler structure.
   a. Series / parallel combination
   b. Voltage / current dividers
   c. Delta-wye conversion
2. Equivalent source is based on Ohm's Law
   a. Thevenin equivalent voltage source
   b. Norton equivalent current source

4.2 Equivalent Impedance Method

Three types of equivalent impedance analysis can be used to reduce problem complexity. These are combining series / parallel impedance, and using ratios of impedance to divide current or voltage, and conversion between delta and wye networks.

4.2.1 Series

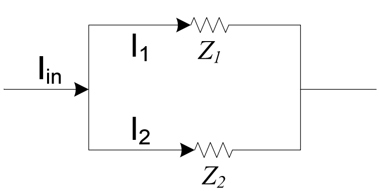


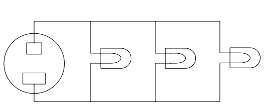


For series impedances, use KVL. The current is the same in each element. So sum the voltages in the branch path.

$$V = V_{Z1} + V_{Z2}$$
$$V_{Z1} = I_1 Z_1$$
$$V_{Z2} = I_1 Z_2$$
$$V = I_1(Z_1 + Z_2) = IZ_{eq}$$
$$\therefore Z_{eq} = Z_1 + Z_2$$

4.2.2 Parallel



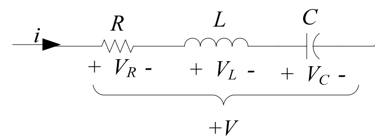


For parallel impedances, use KCL. The voltage is the same across each element. So sum the current into the node.

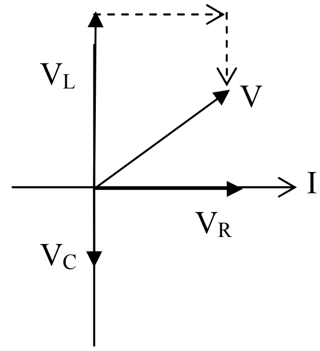
$$I = I_1 + I_2$$
$$I_1 = \frac{V}{Z_1}$$
$$I_2 = \frac{V}{Z_2}$$
$$I = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) = \frac{V}{Z_{eq}}$$

$$\therefore \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

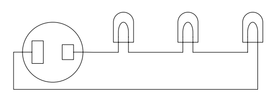


The combination of resistor, inductor, and capacitor can be shown in a phasor of voltages. In series circuits, the current will be the same in all elements, so the impedance is proportional to the voltage. In a parallel circuit the voltage is the same across all elements, so the impedance is proportional to the current.



4.2.3 O, Christmas tree

Christmas tree strings consist of numerous lamps connected in series. The total load is then connected across the power line. As a result, the same current runs through all lamps. If one goes out, all go out.



The voltage rating of each lamp is the total power supply voltage divided by the number of bulbs in the string. Therefore, a bulb from a string of 50 lamps cannot be used in a string of 15 amps.

Many strings now have a thermal contact that is held open by the bulb filament. If the filament fails, the contact will close and leave the remaining lamps energized. Obviously this places less resistance on the line. So the current rises and the voltage drop across each remaining lamp increases. As a result if several bulbs fail, an avalanche effect occurs and the entire string blows out.

4.2.4 Flourescent

Traditionally, most lamps have been incandescent. This is simply a heated element, not much different from the original devices developed by Thomas Edison.

The power rating of lamps is in watts. This is the same rating as heat. Incandescent bulbs convert electrical energy to heat which gives off light. The amount of light created for the heat involved is not very efficient.

Other designs excite gases by various means. The common varieties are fluorescent and metal halides. The excited gases provide much more light for the applied power. Nothing comes free.

The gas excitation requires considerably more voltage than available from the power system. As a result, a transformer is used to step up the voltage. A transformer is an arrangement of two inductors. Because of the inductance, the impedance is complex. The current is out of phase with the voltage. The current increases and the power factor decreases from unity.

$$pf = \frac{P}{VI^*}$$

	EXAMPLES
Ex 1.3-1	Situation: Power supply = 120 Vac, 60 W lamps. What is: current for 1 lamp? $S = P = VI$ $I = \frac{P}{V} = \frac{60W}{120V} = 0.5A$
Ex 1.3-2	What is: impedance for 1 lamp? $Z = \frac{V}{I} = \frac{120V}{0.5A} = 240\Omega$
Ex 1.3-3	Situation: Connect two lamps in parallel. What is: impedance of each lamp? The impedance is basically unchanged by connections. $Z = \frac{V}{I} = 240\Omega$
Ex 1.3-4	What is: voltage on each lamp? Parallel network has the same voltage. $V = 120V$
	What is: current through each lamp? Each has the full voltage, so current is unchanged. $I = 0.5A$
	What is: total current? A parallel circuit sums the current. $\Sigma I = 0.5 + 0.5 = 1.0A$
	What happens to the light intensity? Full current and voltage yields full power. $S = P = 60W$
	Situation: Connect two lamps in series. What is: impedance of each lamp? Impedance is basically unchanged by connections. $Z = 240\Omega$
	What is: voltage on each lamp? Voltage is shared between the two. $V = \frac{120}{2} = 60V$
	What is: total impedance? Total impedance is the sum of series impedance. $Z = 240 + 240 = 480\Omega$

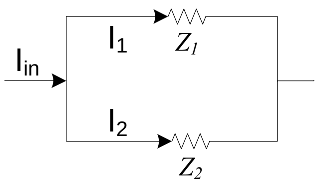
	What is: total current? Total current uses total voltage and impedance. $I = \frac{V}{Z} = \frac{120V}{480\Omega} = 0.25A$
	What is: power on one lamp? Power is voltage across and current through one bulb. $S = P = VI = 60 * 0.25 = 15W$
	What happens to the light intensity from the bulb? Power is reduced by one-fourth, so light is reduced by one-fourth. $P = 15W$
	What happens if one bulb is removed? Removed bulb impedance goes to ∞, so there is no current flow. Both bulbs go out.
	Situation: Power supply = 120 Vac, 60 W lamps, power factor =0.6 for fluorescent fixture. What is: current for 1 lamp? $S = P + jQ$ $P = S\cos\theta = VI\cos\theta$ $I = \frac{P}{V\cos\theta} = \frac{60}{120*0.6} = 0.833A$
	What is: effect of lowering power factor? Increases I, with no increase in work.
	Situation: Power supply = 120 Vac, Christmas tree string with 30 lamps and current of 2A What is: voltage of 1 lamp? $\Sigma V = 120V$ $V = \frac{120}{30} = 4V$
	What is: power of 1 lamp? $S = P = VI = 4 * 2 = 8W$

4.3 Voltage & current dividers

Voltage or current dividers are used to determine voltage across or current through a circuit element. Because of the series and parallel characteristics, proportionality relationships can be established.

4.3.1 Current divider

A current divider is used in parallel circuits with the same voltage applied across the branches.



$$\begin{aligned} I_{in} &= I_1 + I_2 \\ &= \frac{V}{Z_1} + \frac{V}{Z_2} = V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) \\ &= V\left(\frac{Z_1 + Z_2}{Z_1 Z_2}\right) \end{aligned}$$

Rearrange terms.

$$V = I_{in}\left(\frac{Z_1 Z_2}{Z_1 + Z_2}\right)$$

For the parallel circuit, the same voltage is across both impedances.

$$I_1 = \frac{V}{Z_1}$$

Combine the voltage in the two relationships.

$$\begin{aligned} I_1 &= \frac{I_{in}}{Z_1}\left(\frac{Z_1 Z_2}{Z_1 + Z_2}\right) \\ &= I_{in}\left(\frac{Z_2}{Z_1 + Z_2}\right) \end{aligned}$$

A general statement is possible. The current divider is proportional to the opposite impedance over the sum of the impedances.

$$I_1 = \frac{\text{opposite } Z}{\Sigma Z} I_{in}$$

Example

Assume impedance Z_1 is 10Ω and Z_2 is 20 Ω., and the current into the network is 5 Amps.

Find the current in each branch.

Solution:

Find the current in branch 1.

$$\begin{aligned} I_1 &= \frac{\text{opposite } Z}{\Sigma Z} I_{in} \\ &= \frac{Z_2}{Z_1 + Z_2}(I_{in}) \\ &= \frac{20}{10 + 20}(5) = \frac{10}{3} \end{aligned}$$

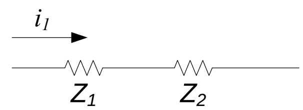
Similarly, the current in branch 2 is determined.

$$I_2 = \frac{\text{opposite } Z}{\Sigma Z} I_{in}$$
$$= \frac{Z_1}{Z_1 + Z_2}(I_{in})$$
$$= \frac{10}{10+20}(5) = \frac{5}{3}$$

In this example, the impedance in branch one is one-half that in branch two. Therefore, the current is twice as great.

4.3.2 Voltage divider

A voltage divider is used in series circuits with the same current in all elements.



$$V_T = V_1 + V_2 \quad \text{from KVL}$$
$$= I\left(Z_1 + Z_2\right)$$

Rearrange terms.

$$I = \frac{V_T}{Z_1 + Z_2}$$

For the series circuit, the current is the same through all impedances.

$$V_1 = IZ_1$$

Combine the current in the two relationships.

$$V_1 = Z_1 \frac{V_T}{Z_1 + Z_2}$$
$$= \frac{Z_1}{Z_1 + Z_2} V_T$$

A general statement is possible. The voltage is proportional to the adjacent impedance over the sum of the impedances.

$$V_1 = \frac{\text{adjacent } Z}{\Sigma Z} V_T$$

The voltage drop is proportional to the impedance.

Example

Assume impedance Z_1 is 10Ω and Z_2 is 20 Ω., and the voltage across the network is 100 Volts.

Find the voltage in each branch.

Solution:

Find the voltage across branch 1.

$$V_1 = \frac{\text{adjacent } Z}{\Sigma Z} V_T$$

$$V_1 = \frac{Z_1}{Z_1 + Z_2} V_T$$
$$= \frac{10}{10+20}(100) = \frac{100}{3}$$

Similarly find the voltage across branch 2.

$$V_2 = \frac{\text{adjacent } Z}{\Sigma Z} V_T$$

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_T$$
$$= \frac{20}{10+20}(100) = \frac{200}{3}$$

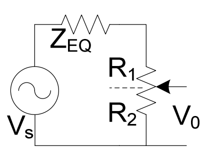
In this example, the impedance in branch one is one-half that in branch two. Therefore, the voltage drop is one-half.

4.4 Applications

4.4.1 Potentiometer

The term potentiometer comes from potential or voltage meter or measurement. A potentiometer or pot is a fixed resistor that has a variable connection. Hence, it is a three terminal network. The output voltage can be adjusted while the load resistance stays fixed. This is a straight forward voltage divider.

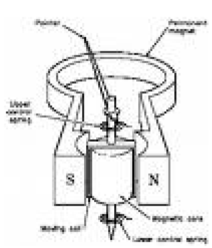
$$V_0 = \frac{R_2}{R_1 + R_2} V_S$$



In some circuits the lower resistance, R_2, is not connected, but is allowed to float. This allows the resistance and resulting current to change on the source. The device becomes a variable resistor.

Potentiometers are commonly used as a volume control on audio or as a dimmer adjustment on lighting. Volume control pots usually rotate about three-fourths of a turn from zero to full scale resistance. Precision potentiometers are multiple turn devices up to 10 turns for full scale.

4.4.2 D'Arsonval meter



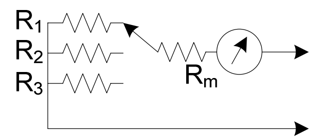
The fundamental non-powered measurement device is an iron-core D'Arsonval meter. A direct current applied to the coil for the meter causes the iron core to rotate due to the magnetic flux. A pointer is connected to the meter and moves proportional to the current.

The meter movement generally responds to 1 milliampere for full scale deflection. Other devices may be as sensitive as 50 microamps for full scale deflection. The internal resistance of the coil is usually in the neighborhood of 50 Ohms.

Since the meter responds to current through the resistance of its winding, it can be treated as any other circuit to determine current or voltage.

Voltmeter

The unit can be applied as a voltmeter by connecting the terminals across the impedance where the voltage is to be measured. The small internal resistance will allow substantial current to flow. It will change the equivalent impedance of the circuit by loading it down.



To use the instrument to measure voltage, a large resistor is connected in series with the meter. A voltmeter is simply a voltage divider.

The size of the series resistor depends on the voltage that is to be measured, the current rating of the meter, and the internal resistance of the meter.

$$Z = R_T = \frac{V_T}{I_M}$$

$$R_S = \frac{V_T}{I_M} - R_M$$

The meter resistance is much less than the series resistance. Therefore, the meter resistance may be ignored for large voltage meters.

The combination with a high resistance improves the sensitivity of the test instrument without significantly impacting the tested circuit. Note that sensitivity is the reciprocal of the current.

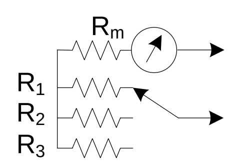
$$Sensitivity = \frac{Z}{V}$$

Ammeter

The same movement can be used to measure current. A very small resistor is placed in parallel with the meter movement. An ammeter is simply a current divider. The shunt resistor is so small that it is often a calibrated length of wire of metal.

$$I_M = \frac{\text{opposite } Z}{\Sigma Z} I_{in}$$

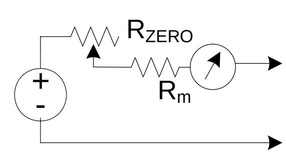
$$Z_P = \frac{I_M}{I_{in} - I_M} Z_M$$



The size of the shunt resistor depends on the current that is to be measured, the current rating of the meter, and the internal resistance of the meter.

Ohmmeter

The meter can only measure voltage or current. Those are two of the three measurement parameters. The third is time. Since all the other electrical parameters are calculated, these can be constructed from the basic meter.



The key component that must be added is a power supply, usually a battery. TO measure inductance or capacitance, an alternating current source is required.

A series resistor must be added to provide the ability to zero the meter as the voltage changes. Short the terminals and adjust the resistor until the meter reads full-scale.

Now the machine is set up to read zero resistance at full-scale. As the resistance of the load increases, the current will decrease. The scale is calibrated to show resistance as the current decreases.

$$I_M = \frac{V_{SUPPLY}}{R_M + R_{ZERO} + R_{TEST}}$$

$$R_{TEST} = \frac{V_{SUPPLY}}{I_M} - (R_M + R_{ZERO})$$

EXAMPLES

Ex 1.3-1

Situation: A potentiometer of 1K with 2 turns full travel is connected across a 5 V signal source.
What is: Voltage for each turn of the pot?

$$R = \frac{1,000\Omega}{2\ \text{turns}} = 500\Omega / turn$$

$$V = \frac{R_1}{R_T} V_s = \frac{500\Omega}{1000}(5) = 2.5V$$

Situation: A D'Arsonval movement rated at 1ma full scale and an internal resistance of 50 Ohms.
What is: series resistance required to measure 1 V, 10 V, and 600 V?

$$R_S = \frac{V_T}{I_M} - R_M$$

$$R_S = \frac{1}{1\times10^{-3}} - 50 = 950\Omega$$

$$R_S = \frac{10}{1\times10^{-3}} - 50 = 9950\Omega$$

$$R_S = \frac{600}{1\times10^{-3}} - 50 = 599,950\Omega$$

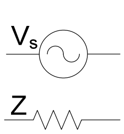
Situation: A D'Arsonval movement rated at 1ma full scale and an internal resistance of 50 Ohms.
What is: shunt resistance required to measure 10 ma, 1A, and 10A?

$$Z_P = \frac{I_M}{I_{in} - I_M} Z_M$$

$$R_P = \frac{1\times10^{-3}}{10\times10^{-3} - 1\times10^{-3}} 50 = \frac{50}{9}$$

$$R_P = \frac{1\times10^{-3}}{1000\times10^{-3} - 1\times10^{-3}} 50 = \frac{50}{999}\Omega$$

$$R_P = \frac{1\times10^{-3}}{10000\times10^{-3} - 1\times10^{-3}} 50 = \frac{50}{9999}\Omega$$

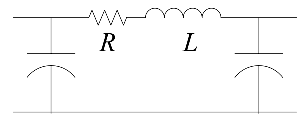


4.5 Two-port networks

Elements and sources are modeled as a one-port network with only two terminals or nodes. All information can be obtained by observing the two terminals.

Most active devices, machines, and networks are modeled as a two-port or four-terminal network. These have two terminals for input and two terminals for output. In some instances, one of the input and output terminals are common. The input port has a voltage and current while the output port has a different voltage and current. The relationship between the input and the output is determined by the configuration of the transfer function between them.

4.5.1 Model comparison



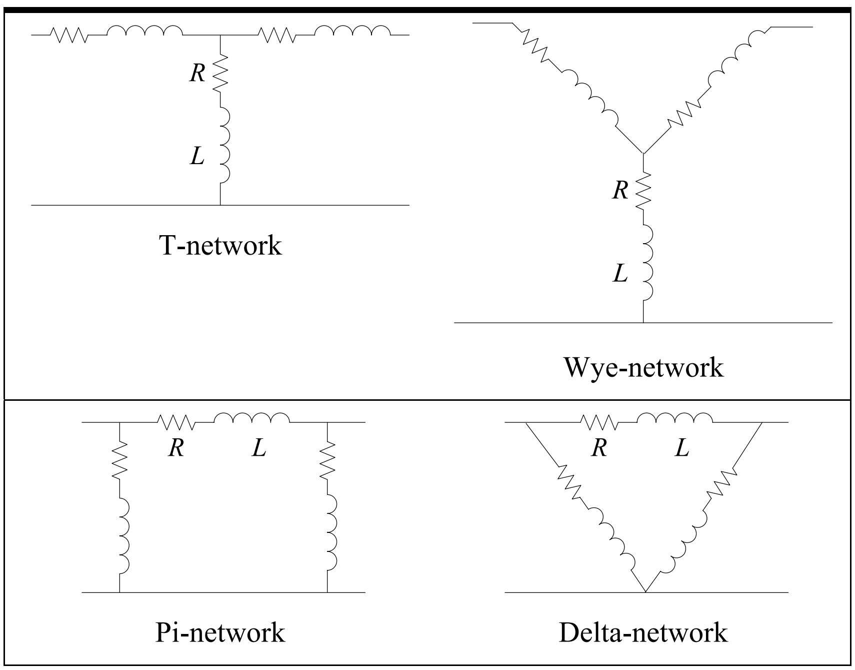
These two-port systems become very common in three-phase alternating current power systems. For a three-phase machine, there are three inductors as elements of the system. These can be connected in only two possible arrangements.

Two-port systems are also used to describe the filters in electronics and controls circuits. They can be a model for a transmission line. Note the same configuration could be used for a transistor or any other network.

The current into the node and the voltage across the terminals determines the transfer relationships. Two-port models have external measurements which take three forms; impedance, admittance, or hybrid. Impedance models have voltage in terms of current. Admittance models have current in terms of voltage. Hybrid models have both voltage and current as outputs.

Circuit	Power
T	Y - wye
Π	Δ - delta
L	L - shunt

The most common arrangement is to use three elements or sources connected in a combination. The result is one side of the two-port network is common. If there is a common internal connection, then there are three possible network arrangements. Although the configuration is the same, different names are used in circuit analysis and power systems.



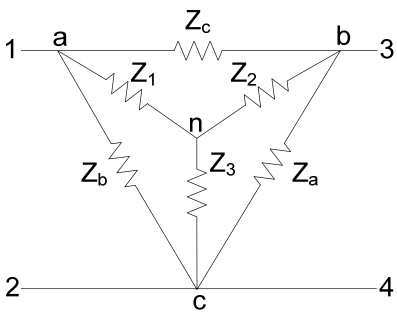
4.5.2 Conversions

In network analysis, the delta or wye connection will often result in a complex arrangement that is not easily solvable. As a result, it is often desired to move from one system to the other.

Conventional Kirchhoff Law analysis could be used, much like in a series / parallel conversion or in a voltage divider. However, it is easier to develop the pattern one time, and then use the pattern to convert between delta (pi) and wye (tee).

Compare the two circuits. Make sure the impedance is equivalent at each node. Apply series and parallel combinations of impedance to obtain the conversion. Consider the connections at the terminals of the two-port system

4.5.2 Delta-wye conversion



$$Z_{12}(Y) = Z_1 + Z_3$$
$$Z_{12}(\Delta) = Z_b \| (Z_a + Z_c)$$

Set the equivalent impedances equal.

$$Z_{12} = Z_1 + Z_3 = \frac{Z_b(Z_a + Z_c)}{Z_a + Z_b + Z_c}$$

A similar process can be conducted for the other two node combinations.

$$Z_{13} = Z_1 + Z_2 = \frac{Z_c(Z_a + Z_c)}{Z_a + Z_b + Z_c}$$

$$Z_{34} = Z_2 + Z_3 = \frac{Z_a(Z_b + Z_c)}{Z_a + Z_b + Z_c}$$

Make substitutions in the three equations to obtain the impedance, Z_1. Subtract Z_{34} from Z_{12}.

$$Z_{12} - Z_{34} =$$
$$Z_1 - Z_2 = \frac{Z_c(Z_b - Z_a)}{Z_a + Z_b + Z_c}$$

Now add the remaining relationship to eliminate Z_2 from the calculation.

$$Z_{12} - Z_{34} + Z_{13} =$$

$$\boxed{Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}}$$

Similar calculations can provide the other two impedances.

$$\boxed{Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}}$$

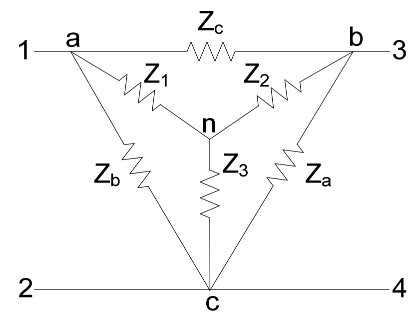
$$\boxed{Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}}$$

A procedure can be described.

Each impedance in a Y-network can be calculated from the product of the two adjacent Δ impedances divided by the sum of the delta impedances.

4.5.3 Wye-delta conversion

A similar process can be used to convert from a wye network to a delta. However, an interaction has been established. Therefore, a combination of the three relationships will be used.



Begin by making a sum of the combinations of two products from the three impedances Z_1, Z_2, and Z_3.

$$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = \frac{Z_aZ_bZ_c\left(Z_a + Z_b + Z_c\right)}{\left(Z_a + Z_b + Z_c\right)^2}$$

Divide the relationship by each of the impedances Z_1, Z_2, and Z_3.

$$\boxed{Z_a = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}}$$

$$\boxed{Z_b = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}}$$

$$\boxed{Z_3 = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}}$$

A procedure can be described.

Each impedance in a Δ-network is the sum of all possible products of Y impedances taken two at a time, divided by the opposite Y impedance.

4.5.4 Balance

When all three impedances in the network are balanced, then the calculations reduce to a very simply relationship.

$$\boxed{Z_Y = \frac{Z_\Delta}{3}}$$

Note that there is no such thing as a three-phase impedance. All impedances must be calculated as phase values.

	EXAMPLES
Ex 1.3-1	Situation: Delta network of transformer at at 60 Hz with inductance of 2,2,and 3 Ohms What is: Wye equivalent network impedance? $Z_1 = \frac{Z_bZ_c}{\Sigma Z}$ $Z_1 = \frac{2*2}{2+2+3} = 4/7$ $Z_2 \& Z_3 = \frac{2*3}{2+2+3} = 6/7$

Ex 1.3-2	Situation: Wye network balanced with 2 Ohm imedance What is: delta equivalent Z? $Z_Y = \frac{Z_\Delta}{3}$ $Z_\Delta = 3Z_Y = 3(2) = 6\Omega$