TRANSFORMER

Transformers consist of two inductors that are closely coupled. Usually an iron core provides an improved magnetic path. Laminations are used in the iron to reduce the hysteresis and eddy current losses. There are no moving parts to a transformer. It simply converts the voltage on one side to a different voltage dependent on the number of turns on each side. The current is converted inversely to the turns.

1. Transformer windings are identified either by location or by terminal markings. Primary windings are labeled with "H". Secondary windings are identified with "X". Subscripts identify the separate terminals.

2. The coupling between the turns is determined by the polarity. Normal polarity is subtractive. The same subscripts are aligned between the primary and secondary. Additive polarity has the opposite subscripts aligned.

3. The voltage (V) ratio between the primary and secondary is equal to the corresponding turns (N) ratio.

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

4. The inverse of the current (I) ratio between the primary and secondary is equal to the turns (N) ratio.

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

5. The same transformer can be used as a step-up or step-down unit. A step-up transformer has a higher voltage and a lower current on the secondary. Conversely, a step-down transformer has a lower voltage and higher current on the secondary.

6. An autotransformer has the secondary and the primary connected together. The voltage is placed on the primary. One terminal becomes common with the output. The other primary terminal is connected to one of the secondary terminals. The remaining secondary terminal becomes the second output terminal. If the secondary is connected with additive polarity, it is a boost connection. If the secondary is connected with subtractive polarity, it is a buck connection.

7. Given: A transformer with a 120 volt primary and a 12 volt secondary.
   Primary current is 10 amps.
   Find: Turns ratio
   Secondary current

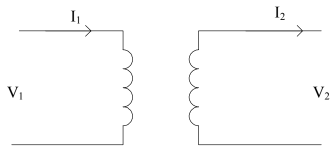
$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

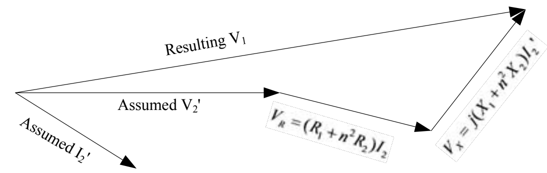
$$\frac{120}{12} = \frac{N_p}{N_s}$$ turns ratio = 10 : 1

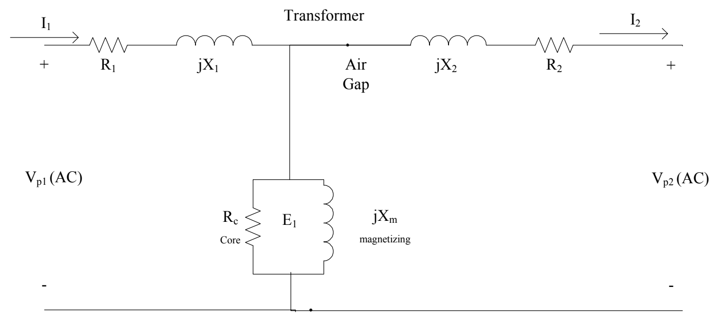
$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

$$\frac{I_s}{10} = \frac{10}{1}$$ $I_s = 10 * 10 = 100$

Transformer







- When Referred to Primary
  - $R_1 = R_p \qquad jX_1 = jX_p$
  - $R_2 = a^2 R_s \qquad jX_2 = ja^2 X_s$
- $V_1 = V_p \qquad V_2 = a\,V_s$
- When Referred to Secondary
  - $R_1 = \dfrac{R_p}{a^2} \qquad jX_1 = \dfrac{jX_p}{a^2}$
  - $R_2 = R_s \qquad jX_2 = jX_s$
  - $R_{core} = R_c / a^2 \qquad X_{core} = j\,X_m / a^2$
- $V_1 = V_p / a \qquad V_2 = V_s$

Voltage Regulation

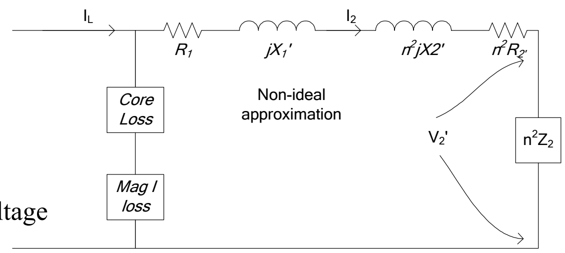
- $V_R = \dfrac{V_{n1-}V_{f1}}{V_{f1}} 100\%$
- $V_{s\ no\ load} = V_p / a$
-

Efficiency

- $\eta = (P_{out} / P_{in}) \times 100\%$
- $\eta = (P_{out} / P_{out} + P_{loss}) \times 100\%$

Transformer Tests

Open Circuit –



- Secondary
- Most voltage drop across excitation coil.
- Shows values of core impedance
- Measure $V_{oc.}$, I_{oc}, P_{oc} with rated primary full voltage applied
- If refer to primary
    - $\frac{1}{Z_E} = Y_E = \frac{1}{R_c} - \frac{j}{X_c}$
    - $|Y_E| = \frac{I_{oc}}{V_{oc}}$
    - $pf = \cos\theta = \frac{P_{oc}}{V_{oc} I_{oc}}$
        - $\theta = \measuredangle Z_E$
        - $-\theta = \measuredangle Y_E$

Short Circuit Test

- Conducted with secondary winding shorted
- Most current flowing is low resistance series path
- Shows values of copper impedance
- Very little current flowing through excitation branch
- $|Z_{SE}| = \frac{V_{sc}}{I_{sc}}$
- $pf = \cos\theta = \frac{P_{sc}}{V_{sc} I_{sc}}$
    - $\theta = \measuredangle Z_E$
- $Z_{SE} = R_{eq} + jX_{eq}$
  $= (R_p + a^2 R_s) + j(X_p + a^2 X_S)$
- Can't separate primary & secondary Z, but usually this is not necessary
- Excitation – core
- Short circuit – copper

Transformer Characteristic Example

Ratings	Open Circuit Values	Short Circuit Values
20 kVA	V=8,000V	V=489V
8000/240V	I=0.214A	I=2.5A
60 Hz	P=400W	P=240W

Tests on primary

1. Open Circuit – find values associated with magnetizing current
   - $$pf = \cos\theta = \frac{P_{oc}}{V_{oc}I_{oc}} = \frac{400}{(8000)(0.214)} = 0.234\,\text{lagging}$$
   - $$Y_E = \frac{I_{oc}}{V_{oc}} \measuredangle -\theta = \frac{0.214}{8000} \measuredangle -76.5^{\circ} = 0.0000063 - j0.0000261\mho$$
   - $$Y_E = \frac{1}{R_c} - j\frac{1}{X_m}$$
     $$\therefore R_c = 159k\Omega, X_m = 38.4k\Omega$$

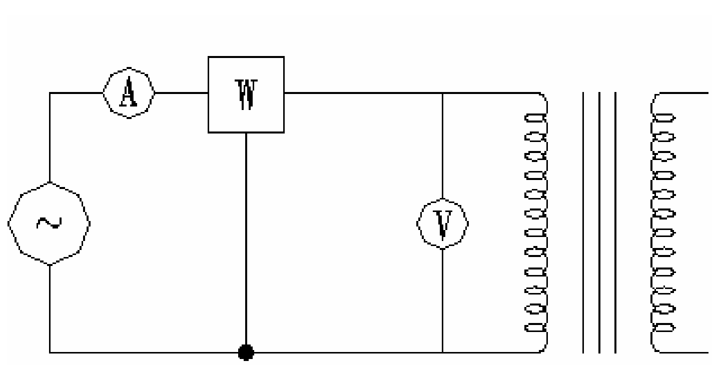
2. Short Circuit – find values associated with series current
   - $$pf = \cos\theta = \frac{P_{sc}}{V_{sc}I_{sc}} = \frac{240}{(489)(7.5)} = 0.196\,lagging$$
   - $$Z_{se} = \frac{V_{sc}}{I_{sc}} \measuredangle \theta = \frac{489}{2.5} \measuredangle 78.7 = 38.4 + j1.92\Omega$$
   - $R_{eq} = 38.4\Omega \quad X_{eq} = 1.92\Omega$
   - These are the values associated with both windings – primary and secondary are not separated.

Transformer Test Connections

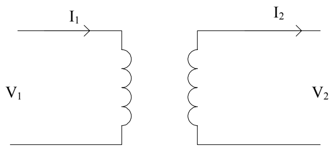
The tests are conducted with the following instrumentation connections. The source is a variac that can adjust the voltage into the transformer. The ammeter and the wattmeter may have a shunt that is used to bypass excessive current.

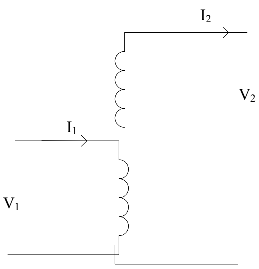
For open circuit, connect as shown. Run at rated voltage.

For short circuit, short the terminals of the transformer. Start at low voltage and increase until near rated current.



AUTOTRANSFORMER





Winding 1 becomes the common coil, N_c, while winding 2 becomes the series coil, N_{se}.

$V_2 / V_1 = N_c / (N_{se} + N_c)$

$I_2 / I_1 = (N_{se} + N_c) / N_c$

The apparent power into and out of the transformer must be equal.

$S \text{ in} = S \text{ out} = S_{io}$

The power in the windings is the same in the common and the series winding

$S_w = V_c I_c = V_{se} I_{se}$

So the ratio of the apparent power gives a “gain” or apparent power advantage.

$S_{io} / S_w = (N_{se} + N_c) / N_c$

Per Unit Notation

- Per unit notation is used to reduce the complexity when working with circuits that have multiple voltage levels.
- Both Ohm's law and the power relationship permit a third term to be calculated from only two terms.
- Two parameters are selected as the reference or base values. These are generally S and V. A different base V is used on each side of a transformer.
- The base current and base impedance can be determined from these two values
  - $I_{base} = \frac{S_{base}}{V_{base}}$
  - $Z_{base} = \frac{V_{base}^{\;2}}{S_{base}}$
- All the circuit equipment voltages and currents are then converted to per unit (percentage) values beore normal circuit calculations are made
  - $S_{pu} = \frac{S_{equip} * 100}{S_{base}}$
  - $V_{pu} = \frac{V_{equip} * 100}{V_{base}}$
  - $I_{pu} = \frac{I_{equip} * 100}{I_{base}}$
  - $Z_{pu} = \frac{Z_{equip} * 100}{Z_{base}}$
- As an example, transformer impedance is usually rated in per unit values. To find the actual impedance, combine the above equations
  - $Z_{equip} = \left(\frac{Z_{pu}}{100}\right) Z_{base}$
  - $Z_{equip} = \left(\frac{Z_{pu}}{100}\right) \frac{V_{base}^{\;2}}{S_{base}}$
- An example illustrates the relationship between per unit values and short circuit capability
  - Transformer, S_{base}=10kVA, V_{base}=120, Z_{pu}=2%
  - $Z_{equip} = \frac{\left(\frac{2}{100}\right) 120^2}{10000} = 0.0288\Omega$
  - $SCC = 10000\left(\frac{100}{2}\right) = 5000kVA$

- $I_{sc} = \frac{V}{Z_{equip}} = \frac{SCC}{V_{base}} = \frac{V_{base}}{Z_{equip}} = 4167A$

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